TALKS

Speaker: Andreas Gross

Title: Tropicalizations of vector bundles and their moduli

Abstract: A naïve way to define tropical vector bundles on rational polyhedral spaces is as torsors over the groups of tropical invertible matrices. This naïve definition can be translated into an equivalent one using line bundles on free covers, which, in turn, leads to a recipe for tropicalizing vector bundles. I will present the challenges that must be overcome to successfully apply this recipe and will show that it can be done for semi-homogeneous vector bundles on maximally degenerating abelian varieties. In this case, which includes all semistable vector bundles on a Tate curve, the image of our tropicalization procedure can be identified with a skeleton of the moduli space of semi-homogeneous vector bundles.
Based on joint work with Martin Ulirsch and Dmitry Zakharov, as well as with Inder Kaur, Martin Ulirsch, and Annette Werner.

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Speaker: Diane Maclagan

Title: Computations with tropical ideals

Abstract: I will describe a new definition, joint with Bivas Khan, for a tropical toric vector bundle on a tropical toric variety. This builds on the tropicalizations of toric vector bundles, and can be used to define tropicalizations of vector bundles on subAvarieties of toric varieties. I will discuss when these bundles do and do not behave as in the classical setting.

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Speaker: Renzo Cavalieri

Title: tropical psi classes and tropicalizations of psi classes

Abstract: the general rule for the interactions between tropical geometry and moduli spaces of course is the following: everything you may wish is going to work like a charm in genus zero, and break down horribly in higher genus. This is the case for the tautological intersection theory of psi classes, a class of fundamental objects in the geometry of moduli spaces of curves: the generating function of their intersection numbers has made waves, pun

intended, when it was noticed that it is a tau function for the KdV hierarchy. Back to tropical geometry: in genus zero tropical psi classes have been first defined by Mikhalkin in the early 2000's, then through the work of Kerber-Markwig and Katz it was shown that intersection numbers of tropical psi classes agree with their algebraic counterparts.
In work with A.Gross and H.Markwig (2021), we were able to make sense of tropical psi classes in higher genus, by making the tropical moduli space of curves into a stack for families of tropical curves with an affine structure. This is a combinatorial theory that recovers the algebraic intersection numbers, but can also produce results that do not have a counter part in algebraic geometry. To this end, in recent work with A.Gross we answer the question of when we can show that tropical psi classes are tropicalizations. In order to even make sense of the statement, we had to introduce a notion of tropicalization for families of curves based on the Picard theory of the base.

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Speaker: Johannes Rau

Title: Universal formulas for quadratically enriched invariants

Abstract: Quadratically enriched invariants as defined in Kass-Levine-Solomon-Wickelgren are values in the Witt-Grothendieck ring obtained by counting rational curves (over an arbitrary field) in del Pezzo surfaces. Specializing these values to rank and signature reproduces the corresponding Gromov-Witten and Welschinger invariants, respectively. Using some floor diagram computations as example, I want to present some conjectures and statements regarding "universal" formulas for these invariants and explain that at least in some cases these formulas are completely determined by certain Welschinger invariants (involving also blow-ups of the original surface). This is ongoing joint work with Erwan Brugallé and Kirsten Wickelgren.

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Speaker: Felipe Rincón

Title: Tropicalising principal minors of positive definite matrices

Abstract: We study the tropicalisation of the image of the cone of positive definite matrices under the principal minor map. This polyhedral complex is a subset of the set of M-concave functions on the discrete n-dimensional hypercube, and we show that it can be characterised as those functions that lie in the tropical flag variety, or equivalently, in a certain slice of the tropical Grassmannian. This is joint work with Abeer Al Ahmadieh, Cynthia Vinzant, and Josephine Yu.

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Speaker :Marvin Hahn

Title: Tropical geometry of b-Hurwitz numbers

Abstract: The Goulden-Jackson b-conjecture is a remarkable open problem in algebraic combinatorics. It predicts an enumerative meaning for the coefficients of the expansion of a certain expression of Jack symmetric functions. Major progress was made in recent work of Chapuy and Dołęga, which led to the introduction of *b-Hurwitz numbers*. These invariants are generalisations of classical Hurwitz numbers obtained by including non-orientable surfaces. They are polynomials in a parameter b, which measures the "non-orientability" of the coverings involved. In this talk, we develop a tropical theory of b-Hurwitz numbers and the recently introduced monotone b-Hurwitz numbers. As part of this development, we resolve an open question posed by Chapuy–Dołęga, as well as an open question posed by Bonzom–Chapuy–Dołęga. This talk is based on joint work in progress with Raphaël Fesler, Maksim Karev and Hannah Markwig.

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Speaker: Johannes Schmitt

Title: Logarithmic tautological rings

Abstract: Given a smooth space X with normal crossings divisor D, the logarithmic Chow ring of (X,D) describes the Chow rings of of all iterated blow-ups of boundary strata of D simultaneously. Its applications range from logarithmic double ramification cycles (and associated enumerative invariants like double Hurwitz numbers) to the intersection theory of toroidal compactifications of moduli spaces of abelian varieties. In my talk, I explain how piecewise polynomials on the tropicalization of (X,D) can be used to construct natural log Chow classes and describe work with Pandharipande, Ranganathan and Spelier on defining the logarithmic tautological ring of (X,D).

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Speaker: Luca Schaffler

Title: An explicit wall crossing for the moduli space of hyperplane arrangements

Abstract: The moduli space of hyperplanes in projective space has a family of geometric and modular compactifications that parametrize stable hyperplane arrangements with respect to a weight vector. Among these, there is a toric compactification that generalizes the Losev-Manin moduli space of points on the line. We study the first natural wall crossing that modifies this

compactification into a non-toric one by varying the weights. As an application of our work, we show that any Q-factorialization of the blow up at the identity of the torus of the generalized Losev-Manin space is not a Mori dream space for a sufficiently high number of hyperplanes. Additionally, for lines in the plane, we provide a precise description of the wall crossing. This is joint work with Patricio Gallardo.

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Speaker: Yoav Len

Title**:** The geometry of Prym varieties

Abstract**:** I will talk about Prym varieties, a generalisation of Jacobians that shows up when studying symmetries of curves and graphs. Pryms have deep connections with tangent lines of curves, spin structures, and torsion points of Jacobians. Constructing and compactifying the moduli space of these objects is a notoriously difficult problem. Nevertheless, I will discuss joint work with Poiret, in which we use tropical geometry to construct a universal space for Prym varieties. Furthermore, I will describe ongoing work with Molcho and Nabijou towards determining the geometry of the Prym locus within the moduli space of abelian varieties.

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Speaker: Lionel Lang

Title: Logarithmic volumes of holes of hypersurfaces and tropicalisation of periods.

Abstract: Integrating the logaritmic volume form on well chosen discs bounded on hypersurfaces gives a local system of coordinates on the linear system of such hypersurfaces. Surprisingly, the tropical analogue of this procedure gives global coordinates on the corresponding linear system of tropical hypersurfaces. In both the algebraic and tropical settings, the differential of these coordinate systems with respect to the coefficents of the defining equations are period matrices. In this talk, I want to discuss how the latter observation can be used to study the tropicalisation of periods of hypersurfaces.

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Speaker: Ilia Itenberg

Title:Real enumerative invariants relative to the toric boundary

Abstract: The talk is devoted to several real and tropical enumerative problems. We suggest new real enumerative invariants of the projective plane (and, more

generally, of certain toric surfaces) that arise from appropriate signed enumeration of real algebraic curves of genus 1 and 2. It turns out that two different rules of signs in the enumeration lead to the same collection of invariants. The proof of this surprising fact uses the tropical counterparts of the invariants under consideration. This is a joint work with Eugenii Shustin.

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Speaker: Luca Battistella

Title: Chow rings of moduli of Gorenstein curves of genus one with at most six markings

Abstract: Modular compactifications of $M_{1,n}$ parametrising only Gorenstein curves have been constructed and classified by Smyth and Bozlee-Kuo-Neff (there are a lot of them). The combinatorially simplest one $U_{1,n}$ parametrises curves without rational tails; for $n \leq 6$, Lekili and Polishchuk identified it with a weighted projective stack or a Grassmannian. In joint work with Andrea Di Lorenzo, we consider the Artin stack $G_{1,n}$ of log-canonically polarised Gorenstein curves of genus one. It admits a stratification by tail type, whose strata are products of $U_{1,m}, m \leq n$ with moduli spaces of stable rational curves. For $n \leq 6$, we find an explicit description of its integral Chow ring by patching. The Chow ring of any modular compactification (including $\overline{M}_{1,n}$) can be obtained from $A^*(G_{1,n})$ by excision. Moreover, these spaces satisfy the Chow-Künneth generation property (implying rational Chow=cohomology and polynomial point count).

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Speker: Thomas Blomme

Title: A short proof of the multiple cover formula

Abstract: Enumerating genus g curves passing through g points in an abelian surface is a natural problem, whose difficulty highly depends on the degree of the curves. For "primitive" degrees, we have an easy explicit answer. For "divisible" classes, such a resolution is quite demanding and often out of reach. Yet, the invariants for divisible classes easily express in terms of the invariants for primitive classes through the multiple cover formula, conjectured by G. Oberdieck a few years ago. In this talk, we'll show how tropical geometry enables to prove the formula without any kind of concrete enumeration.

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Speaker : Dan Abramovich

Title : Normal-crossings resolution using weighted blowups

Abstract : Recent work shows how to resolve singularities in characteristic 0 using weighted blowups, but does not make the exceptional a normal crossings divisor, a requirement of applications.
We show how "sufficiently strong resolution" proves "equally strong normal-crossings resolution". This is joint work with Belotto da Silva, Quek, Temkin and Wlodarczyk.

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Speaker : Margarida Melo

Title : Compactified spaces of roots over the space of curves.

Abstract : Given a line bundle L over the moduli space of curves, the space parametrizing r-th roots of L yields a natural finite cover of the space of curves. Spaces of roots are very interessing as they carry lots of geometrical information on the spaces of curves themselves and are interesting from the point of view of enumerative Geometry.
Starting from the well known case of spin curves, we will discuss how to study nice compactifications of these spaces over the moduli space of stable curves.

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Speaker : Jonathan Wise

Title : The Rhizomic Topology

Abstract : The logarithmic étale topology permits a logarithmic scheme to see the topology of a nearby family that has been punctured along the support of the logarithmic structure. This remarkable property allows a logarithmic scheme to see the nearby fiber of a degeneration, without requiring and explicit construction of the degeneration. It also permits the local representability of moduli problems that are not locally representable in the strict étale topology. However, the logarithmic étale topology is well known not to be subcanonical, and therefore is not suitable for descent. I will describe a coarsening of the logarithmic étale topology that remedies some of its demerits while maintaining its useful features.

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Short Talks

Speaker: Alessio Cela

Title: Brill-Noether Theorem for (toric) surfaces

Abstract: The classical Brill-Noether Theorem states that every non-degenerate map from a general genus g curve C to projective space is a point of the expected dimension in the moduli space of such maps. In this talk, I will present an analogous statement for maps from C to smooth projective surfaces. In the case of toric surfaces, I will provide a geometric interpretation of what it means for a map to be non-degenerate. This is joint work with Carl Lian.

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Speaker: Arne Kurs
Title: Tropical Prinicipal Bundles on Metric Graphs

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Speaker: Siao Chi Mok

Title: Logarithmic Fulton—MacPherson configuration spaces

Abstract**:** The Fulton—MacPherson configuration space is a well-known compactification of the ordered configuration space of a projective variety. We extend the construction to the logarithmic setting: it is a compactification of the configuration space of points on a projective variety X away from a simple normal crossings divisor D, and is constructed via logarithmic geometry. This new construction enables us to make sense of a Fulton—MacPherson space of a non-compact variety. Moreover, given a simple normal crossings degeneration of X, logarithmic geometry similarly enables a degeneration of the Fulton—MacPherson space of X. Time permitting, we will see some potential applications of these constructions.

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Speaker: Giusi Capobianco

Title: Weighted harmonic double covers and the tropical Abel-Prym map

Abstract : In this talk I will recall the notion of weighted harmonic double cover of metric graphs and focus on the hyperelliptic case. I will define the tropical Abel-Prym map and give some results concerning the degree of this map. This

is part of the ongoing work for my thesis project.

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Speaker: Terry Song

Title: Dual complex of genus one mapping spaces

Abstract: The dual complex of a smooth variety encodes the combinatorial structure that underlies all its possible normal crossings compactifications. We describe the dual complexes of genus zero and genus one mapping spaces as moduli spaces of decorated tropical curves and prove that are contractible (in degrees > 0 and > 1 respectively) via an explicit deformation retraction. In genus one, the key geometric input comes from the Vakil-Zinger space and its logarithmic interpretation due to Ranganathan - Santos-Parker - Wise. Joint work with Siddarth Kannan (MIT).

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Speaker: Cat Rust

Title: Expanded Stable Maps to the Logarithmic Torus

Abstract**:** We will begin with the stratification by combinatorial types of the moduli space M(**x**) of expanded stable maps to the logarithmic torus with tangencies given by **x**. We will then classify 'generic' and 'non-generic' combinatorial types and see how this leads us to split the tangency space up into regions where χ(M(x)) is constant (work in progress.)
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Speaker: Pim Spelier

Title: TBA

Abstract: TBA

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Speaker : Tangi Pasquer

Title: Refined tropical invariants and upper bounds on numbers of real curves.

Abstract: Tropical geometry can be used to compute enumerative invariants, e.g the number of complex curves of genus g and degree d through 3d+g-1 generic points on the plane. G. Mikhalkin's work from 2003 provides an

influential realization of this principle, introducing multiplicities for tropical curves satisfying analogous conditions to count complex and real curves. Refined versions of these multiplicities were later proposed by F. Block & L. Göttsche, the invariance of which has been proved by I. Itenberg & G. Mikhalkin in 2012. In their article, they also deduce an interesting consequence: there exist choices of 3d+g-1 generic real points with nonreal genus g, degree d curves passing trough, for sufficiently large d.