Integrability in Condensed Matter Physics and Quantum Field Theory

Conference in Les Diablerets, Suisse, February 3-12, 2023

The Loom for Generalized Fishnet CFTs

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Review and work with Enrico Olivucci Arxiv:2212 .09732

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LPENS LABORATOIRE DE PHYSIQUE DE L'ÉCOLE NORMALE SUPÉRIEURE

Outline

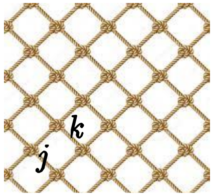
- Fishnet CFT was discovered as a special double scaling limit of γ-deformed $\mathcal{N}$=4 SYM : strong imaginary γ-deformation & weak coupling

Gurdogan, V.K. '15

- Integrability of planar $\mathcal{N}$=4 SYM theory becomes in this limit manifest and allows to compute many non-trival quantities: anomalous dimensions, amplitudes, some structure constants…

- Fishnet CFT dominated by planar graphs with a regular lattice structure – fishnets: Integrable via SO(2,D) conformal spin chain

$$\int \prod_i d^D x_i \prod_{<jk>} \frac{1}{|x_j - x_k|^{D/2}}$$



A. Zamolodchikov 1980

- A. Zamolodchikov proposed a general construction of integrable planar graphs, based on Baxter lattice and star-triangle relations

- We propose generalized Fishnet CFTs at any D, generated by any such Baxter lattice

γ-twisted N=4 SYM and "fishnet" limit

ϕ_1
ϕ_2
ϕ_3
ψ_1
ψ_2
ψ_3
λ
A

$$\mathcal{L} = N_c \mathrm{tr}\Big(F^2 + D\bar\phi_i D\phi_i + i\bar\psi_j \not{D}\psi_j + i\bar\lambda\not{D}\lambda +$$
$$+ g^2 [\phi_j,\phi_k]_q \cdot [\bar\phi_j,\bar\phi_k]_q + i\, g\, \epsilon_{ijk}\bar\psi_k[\phi_i,\bar\psi_j]_q + g\,\bar\lambda[\phi_j,\bar\psi_j]_q + \text{ conj. }\Big)$$

- γ-twisted N=4 SYM Lagrangian: commutators → q-commutators

Leigh, Strassler
Frolov, Tseytlin
Beisert, Roiban
Lunin, Maldacena

$$[A\,,B] \to [A,B]_q \equiv q_{AB}\,A\,B - \frac{1}{q_{AB}}\,B\,A$$

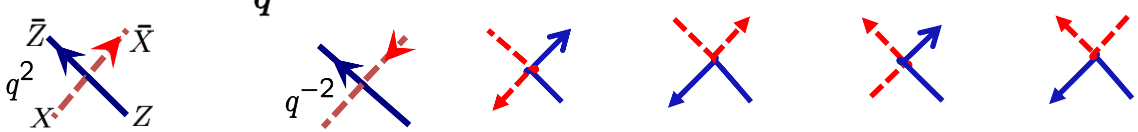
where $q_{A,B} = e^{-\frac{i}{2}\epsilon^{mjk}\gamma_m\,J^A_j\,J^B_k} = (q_{B,A})^{-1}$

$\gamma_1, \gamma_2, \gamma_3$ - twists $\qquad J^A_1, J^A_2, J^A_3 \in SO(6)$ - Cartan charges of R-symmetry

- Breaks R-symmetry and all supersymmetry: PSU(2,2|4)→SU(2,2) × $U(1)^3$

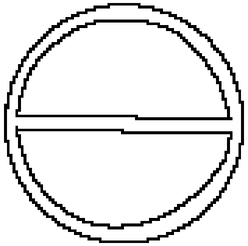
4-scalar term $\quad \frac{g^2}{16}\mathrm{tr}\,([X,Z]_q[\bar X,\bar Z]_q) =$ $\qquad \phi_1 \equiv X, \quad \phi_2 \equiv Z$

$$= g^2 \mathrm{tr}\Big(2q^2\, ZX\bar Z\bar X + 2\frac{1}{q^2}\, Z\bar X\bar Z X - Z\bar X X\bar Z - Z\bar Z X\bar X - ZX\bar X\bar Z - Z\bar Z\bar X X\Big) + \ldots \qquad q = e^{\frac{i}{2}\gamma_3}$$

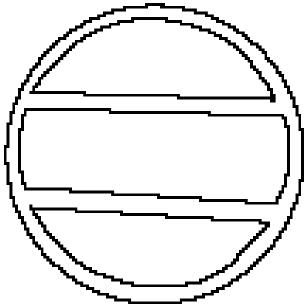


- γ-twist is a topological factor on a planar graph:
  it produces quasiperiodic b.c. on cylindric graph

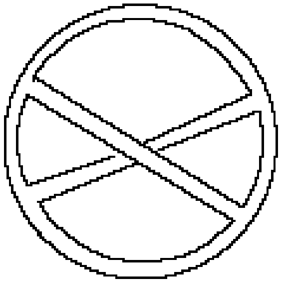
‘t Hooft planar limit $N_c \to \infty$



$$= \lambda^2 N^{2-3+3} = \lambda^2 N^2$$



$$= \lambda^4 N^{4-6+4} = N^2 \lambda^4$$



$$= \lambda^4 N^{4-6+2} = N^0 \lambda^4$$

Planar graphs have maximal number of faces (for the same number of vertices.

Planar graphs can be drawn on the sphere without slf-intersections.

Chiral CFT and Dynamical “Fishnet”

Gurdogan, V.K. ‘15

- Double scaling “fishnet” limit: Strong imaginary twist, weak coupling:

$$g \to 0, \qquad \gamma \to i\infty, \qquad \xi_j = g\, e^{-i\gamma_j/2} - \text{fixed}, \qquad (j = 1, 2, 3.)$$

- Chiral CFT from double-scaled γ-twisted N=4 SYM:

$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{2}\partial^\mu \bar\phi_i \partial_\mu \phi^i + i\bar\psi^{\dot\alpha}_A \partial^\alpha_{\dot\alpha} \psi^A_\alpha] + \mathcal{L}_{\text{int}}$$

3 flavors of bosons and fermions

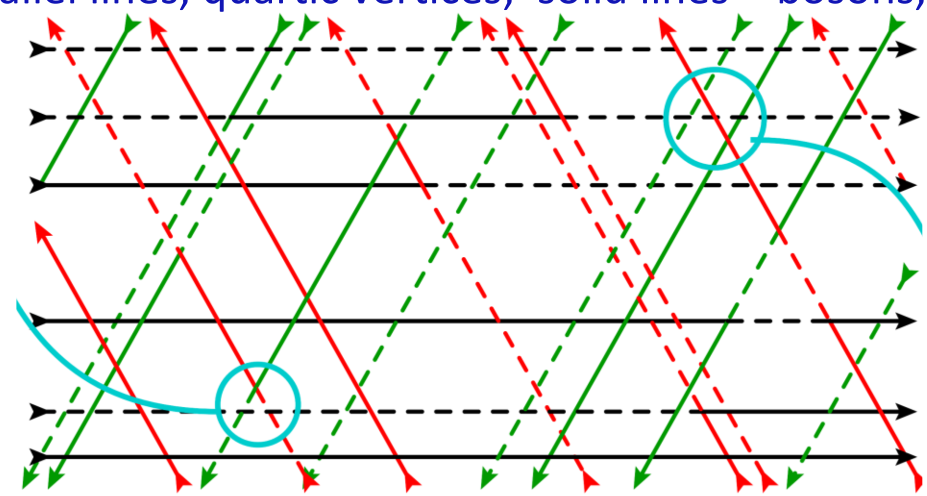
$$\mathcal{L}_{\text{int}} = N_c\, \mathrm{tr}[\xi_1^2\, \bar\phi_2\bar\phi_3\phi_2\phi_3 + \xi_2^2\, \bar\phi_3\bar\phi_1\phi_3\phi_1 + \xi_3^2\, \bar\phi_1\bar\phi_2\phi_1\phi_2 +$$

$$+ i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2 + \bar\psi_3\bar\phi_1\bar\psi_2) + i\sqrt{\xi_1\xi_3}(\psi^1\phi^2\psi^3 + \bar\psi_1\bar\phi_2\bar\psi_3) + i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1 + \bar\psi_2\bar\phi_3\bar\psi_1)].$$

- Planar Feynman graphs form a dynamical fishnet:

3 systems of parallel lines, quartic vertices; solid lines – bosons, dotted lines - fermions

V.K., Olivucci, Preti 2018



Intersection with fermionic lines should be disentangled into two Yukawa vertices

A challenge: to uncover the underlying integrable spin chain.

A step towards understanding of full N=4 SYM integrability.

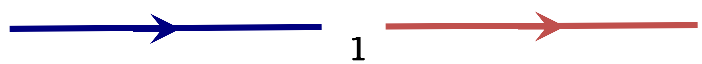
Gurdogan, V.K. 2015

Special case: bi-scalar Fishnet CFT_4

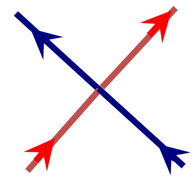
- Only one double-scaling coupling turned on

$$\mathcal{L}[\phi_1,\phi_2] = \frac{N_c}{2}\mathrm{tr}\left(\partial^\mu\bar\phi_1\partial_\mu\phi_1 + \partial^\mu\bar\phi_2\partial_\mu\phi_2 + 2\xi^2\,\bar\phi_1\bar\phi_2\phi_1\phi_2\right).$$

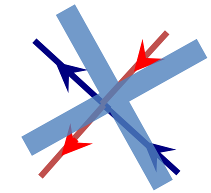
- Propagators



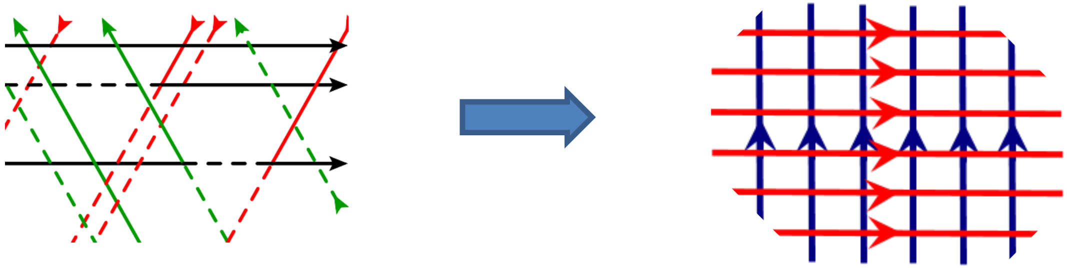
$\sim \frac{1}{(x-y)^2}$

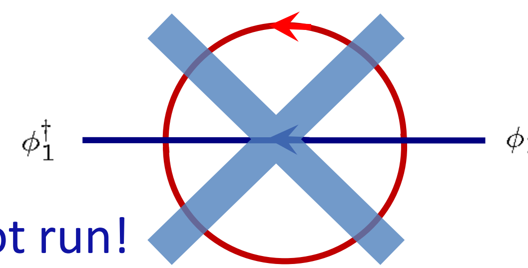


Missing “anti-chiral” vertex



- $\mathcal{N}$= 4 SYM planar graphs reduce, in the bulk, to (very few!) fishnet graphs

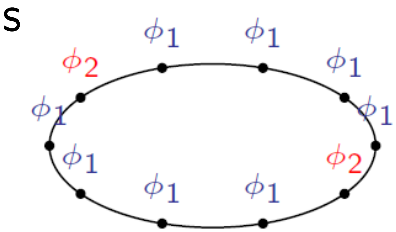




- No mass or vertex renormalization! Coupling ξ does not run!

- Local operators

$$\mathcal{O}(x) = C^{\mu_1\cdots\mu_n}\,\mathrm{tr}\left[\partial_{\mu_1}\ldots\partial_{\mu_n}(\phi_1)^L(\phi_2)^M(\bar\phi_1)^K(\bar\phi_2)^N\right](x) + \text{permutations}$$



anomalous dimension

- Correlators $\langle\mathcal{O}(x)\ \mathcal{O}(0)\rangle \sim |x|^{-2\Delta_0-2\gamma(\xi)}$

V.K., Olovucci '22

"PT"-invariance and reality of spectrum

T-transformation

$$\mathcal{L}(\phi_1, \phi_2) \quad \rightarrow \quad \overline{\mathcal{L}(\phi_1, \phi_2)}$$

"P"-transformation (transpose)

$$\phi_1 \quad \rightarrow \quad \phi_1^t \,, \qquad \phi_2 \quad \rightarrow \quad \phi_2^t$$

PT-transformation leaves the action invariant:

$$\mathrm{tr}(\phi_1\phi_2\bar{\phi}_1\bar{\phi}_2) \quad \overset{T}{\rightarrow} \quad \mathrm{tr}(\phi_2\phi_1\bar{\phi}_2\bar{\phi}_1) \quad \overset{\text{``}P\text{''}}{\rightarrow} \quad \mathrm{tr}(\phi_1\phi_2\bar{\phi}_1\bar{\phi}_2)$$

Operators get in general transformed, e.g.

$$\mathrm{tr}(\phi_1\phi_1\phi_2\bar{\phi}_1) \quad \overset{PT}{\rightarrow} \quad \mathrm{tr}(\bar{\phi}_1\bar{\phi}_1\bar{\phi}_2\phi_1)$$

Conformal dimension gets complex conjugate (non-unitary theory!):

$$\left[\langle\bar{\mathcal{O}}(x)\mathcal{O}(0)\rangle\right]^{\mathrm{PT}} = \langle\bar{\mathcal{O}}^{\mathrm{PT}}(x)\mathcal{O}^{\mathrm{PT}}(0)\rangle = |x|^{-2\Delta^*}$$

The spectrum consists of real dimensions or of complex conjugate pairs!

Similar to non-unitary PT-invariant quantum mechanics

Bender & Boettcher '98

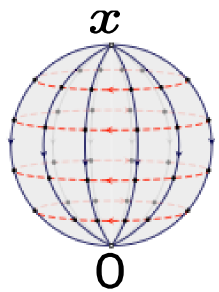
$$\mathcal{H} = \hat{p}^2/2 + x^2\,(ix)^{\epsilon}$$

Caetano, Gurdogan, V.K , 2016

Operators, correlators, graphs…

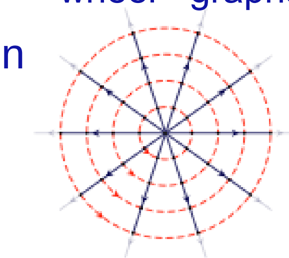
$\mathrm{tr}[\phi_1(x)]^L$

BMN "vacuum"
(non-protected)



UV- reduction

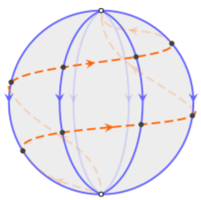
"wheel" graphs - divergent! Need $\epsilon = 4 - D$ regularization



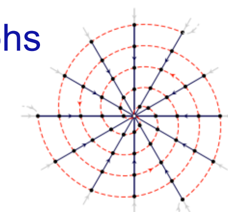
$$= \xi^{2LM}\left(\frac{\#}{\epsilon^M} + \frac{\#}{\epsilon^{M-1}} + \ldots + \frac{C_M}{\epsilon} + \text{finite}\right)$$

$$\gamma(\xi) = \sum_{M=1}^{\infty} C_M\, \xi^{2LM}$$

Multi-magnon
spiral graphs



"spiderweb" graphs

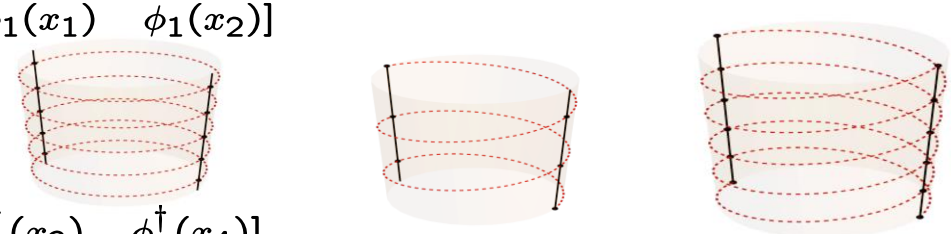


Typical "fishnet" structures
in the bulk of graphs. Integrable!

Gurdogan, V.K. 2015
Caetano, Gurdogan, V.K. 2016
Basso, Zhong '19
Basso, Ferrando, V.K., Zhong '19

- L=2 4-point functions, explicit computations

$\mathrm{tr}[\phi_1(x_1)\quad \phi_1(x_2)]$

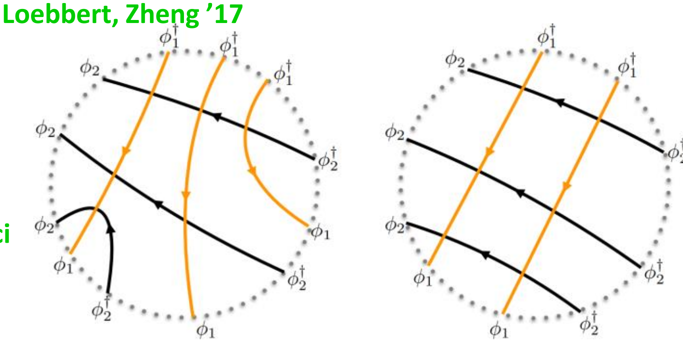


$\mathrm{tr}[\phi_1^\dagger(x_3)\quad \phi_1^\dagger(x_4)]$

Grabner, Gromov, V.K , Korchemsky '17
V.K., Olivucci 2018
Gromov, V.K , Korchemsky '18
V.K., Olivucci, Preti, '19
Gromov, Sever '20

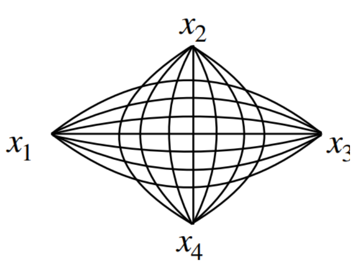
- Amplitudes, Yangian symmetry

Chicherin, V.K., Mueller, Loebbert, Zheng '17
Korchemsky '19
Loebbert et al '20'21'22'
Duhr et al '22



- Basso-Dixon-type 4-point functions

$G_{m,n}(x_1, x_2, x_3, x_4) =$

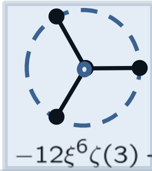


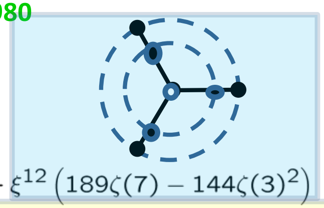
Basso, Dixon
Derkachev, V.K., Olivucci
Derkachev, Ferrando, Olivucci
Dercachov, Olivucci
Kostov '23
…

Gromov, V.K , Korchemsky, Negro, Sizov (2017)

Dimension of $\mathrm{tr}(\phi_1)^3$ and periods of wheel graphs from QSC

Broadherst 1980



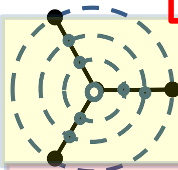


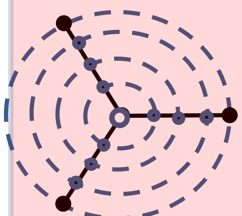
Ahn, Bajnok, Bombardelli, Nepomechie 2013

E.Panzer, 2015

Gurdogan, V.K. '15 (any number of spokes)

In terms of Riemann (multi)-zeta numbers

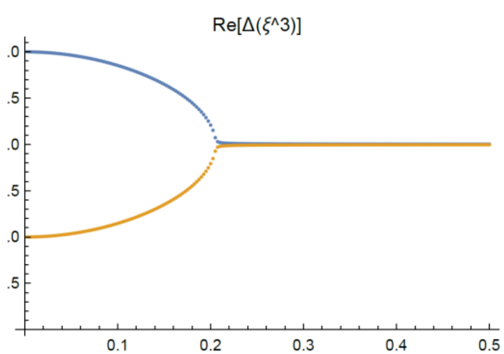


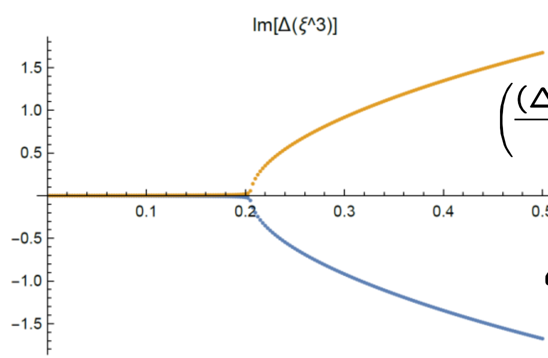


$$
\begin{aligned}
\Delta - 3 = & -12\xi^6\zeta(3) + \xi^{12}\left(189\zeta(7) - 144\zeta(3)^2\right) \\
& +\xi^{18}\Big(-1944\zeta(8,2,1) - 3024\zeta(3)^3 - 3024\zeta(5)\zeta(3)^2 + \frac{198\pi^8\zeta(3)}{175} + 6804\zeta(7)\zeta(3) \\
& + \frac{612\pi^6\zeta(5)}{35} + 270\pi^4\zeta(7) + 5994\pi^2\zeta(9) - \frac{925911\zeta(11)}{8}\Big) + \\
& \xi^{24}\Big(\frac{10368}{5}\pi^4\zeta(8,2,1) + 5184\pi^2\zeta(9,3,1) + 51840\pi^2\zeta(10,2,1) - 148716\zeta(11,3,1) \\
& -1061910\zeta(12,2,1) + 62208\zeta(10,2,1,1,1) - 93312\zeta(3)\zeta(8,2,1) - 288\zeta(3)^5 \\
& +72\gamma\pi^2\zeta(3)^4 - 77760\zeta(3)^4 - \frac{80756\pi^6\zeta(3)^3}{945} - 145152\zeta(5)\zeta(3)^3 - \frac{29}{270}\gamma\pi^8\zeta(3)^2 \\
& + \frac{9504\pi^8\zeta(3)^2}{175} - 879\pi^4\zeta(5)\zeta(3)^2 - 2025\pi^2\zeta(7)\zeta(3)^2 + 244944\zeta(7)\zeta(3)^2 \\
& +186588\zeta(9)\zeta(3)^2 + \frac{2910394\pi^{12}\zeta(3)}{2627625} - 2592\pi^2\zeta(5)^2\zeta(3) + \frac{29376}{35}\pi^6\zeta(5)\zeta(3) \\
& +12960\pi^4\zeta(7)\zeta(3) + 298404\zeta(5)\zeta(7)\zeta(3) + 287712\pi^2\zeta(9)\zeta(3) \\
& -5555466\zeta(11)\zeta(3) + 57672\zeta(5)^3 - 71442\zeta(7)^2 + \frac{13953\pi^{10}\zeta(5)}{1925} + \frac{7293\pi^8\zeta(7)}{175} - \frac{19959\pi^6\zeta(9)}{5} \\
& + \frac{119979\pi^4\zeta(11)}{2} + \frac{10738413\pi^2\zeta(13)}{2} - \frac{4607294013\zeta(15)}{80}\Big) + O\left(\xi^{25}\right)
\end{aligned}
$$

- Numerics of high precision:

Gromov, V.K , Korchemsky, Negro, Sizov (2017)





Baxter eq.

$$\left(\frac{(\Delta-1)(\Delta-3)}{4u^2} - \frac{i\,\xi^3}{u^3} - 2\right) q(u) + q(u+i) + q(u-i) = 0$$

Quantization condition

$$q_2(0,\xi)\, q_4(0,-\xi) + q_2(0,-\xi)\, q_4(0,\xi) = 0$$

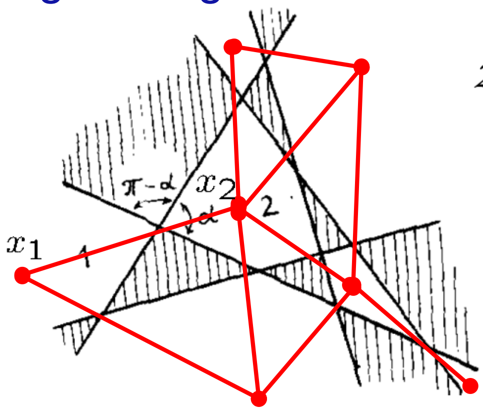
- Generalization to any number of spokes&magnons possible

Gromov, Sever '19

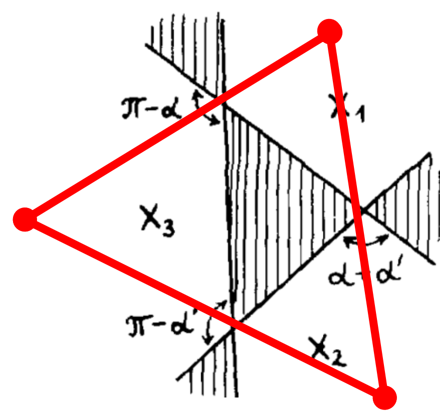
A. Zamolodchikov "Fishnet" graph Integrability

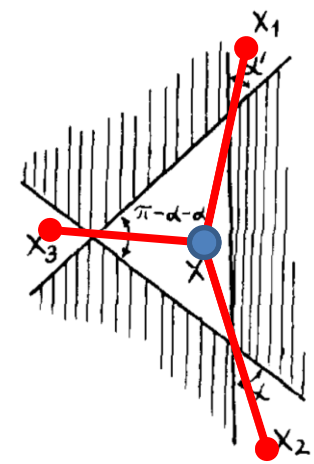
- Feynman is graph dual to Baxter lattice (intersecting straight lines on the plane)
- Dash all the faces connected through the common vertices forming sublattice type I, leaving blank similar complimentary sub-lattice of type II.
- D-dimensional integration variable x_j in the middle of each blank face
- Neighboring vertices connected by propagators

$$G_D(x_j, x_k, \alpha_{jk}) = |x_j - x_k|^{\frac{D}{\pi}(\alpha_{jk}-\pi)}$$



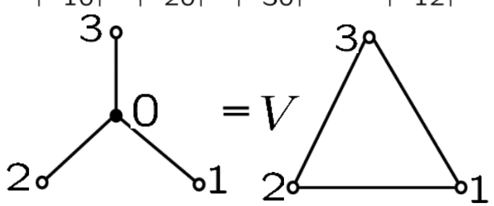
$$\mathcal{Z}_B = \int \prod_{m\in\mathcal{L}_I} d^D x_m \prod_{<j,k>\in\mathcal{L}_I} G_D(x_j, x_k, \alpha_{jk})$$





- We can move a line past intersection due to star-triangle relation (a version of Yang-Baxter relation):

$$\int \frac{d^D x_0}{|x_{10}|^{2a}|x_{20}|^{2b}|x_{30}|^{2c}} = \frac{V(a,b,c)}{|x_{12}|^{D-2c}|x_{23}|^{D-2a}|x_{31}|^{D-2b}}, \qquad (a+b+c = D, \quad x_{ij} := x_i - x_j)$$



if we choose angles as $a = \frac{D}{\pi}\alpha, \quad b = \frac{D}{\pi}\beta, \quad c = \frac{D}{\pi}\gamma$

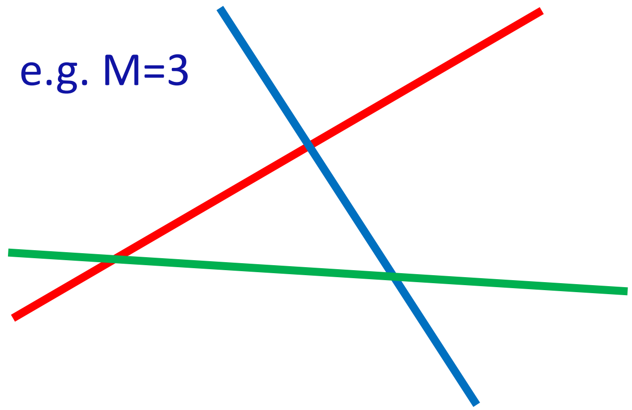
$$V(a,b,c) = \pi^{D/2}\frac{\Gamma(\frac{D}{2}-a)\Gamma(\frac{D}{2}-b)\Gamma(\frac{D}{2}-c)}{\Gamma(a)\Gamma(a)\Gamma(c)}$$

V.K., Olovucci '22

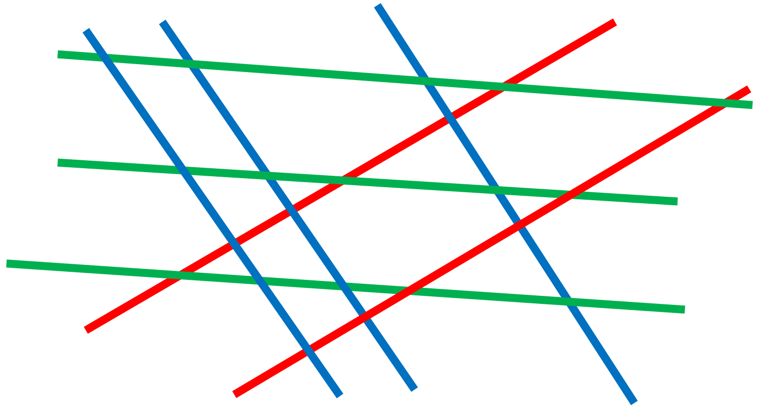
Loom for fishnet CFTs from Baxter lattices

- Baxter lattice for general Fishnet CFT: M intersecting lines with M slopes

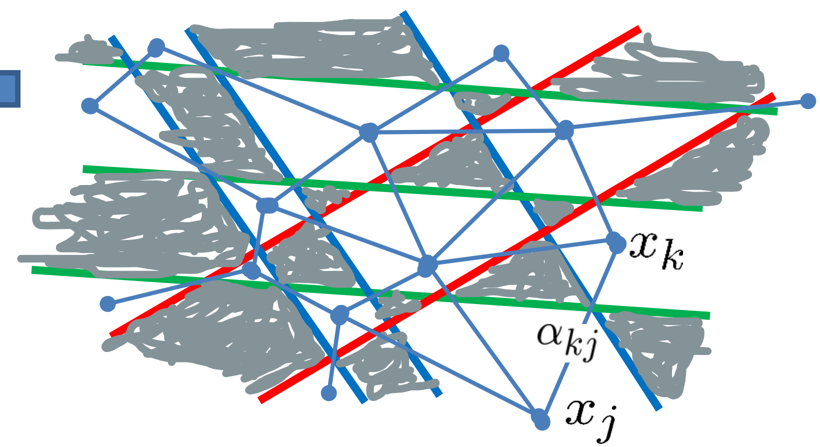
e.g. M=3



add parallel lines at any positions (loom)



Connect the vertices at even (white) faces by appropriate propagators



Construct Fishnet CFT$^{(M)}$ with all such Feynman graphs (related by star-triangle)

$$\mathcal{G}_B = \int \prod_{m\in\mathcal{L}_I} d^D x_m \prod_{<j,k>\in\mathcal{L}_I} G_D(x_j, x_k, \alpha_{jk})$$

$$G_D(x_j, x_k, \alpha_{jk}) = |x_j - x_k|^{\frac{D}{\pi}(\alpha_{jk}-\pi)}$$

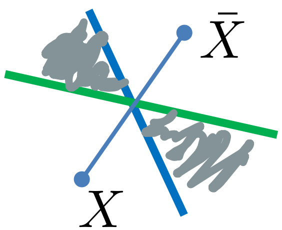
Alfimov, V.K., Olivucci '22

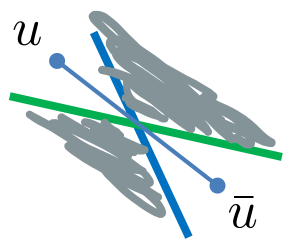
Generalized Fishnet CFT: Kinetic Terms

All Feynman graphs of the loom are connected by star-triangle relations.
To accommodate all these graphs within a Fishnet CFT$^{(M)}$, we need *M(M-1)* scalar fields (two for each crossing)

Example of *M=3*:
6 fields with dimensions

$$\Delta_X = a$$
$$\Delta_Y = b$$
$$\Delta_Z = \frac{D}{2} - a - b$$
$$\Delta_u = \frac{D}{2} - a$$
$$\Delta_v = \frac{D}{2} - b$$
$$\Delta_w = a + b$$





Kinetic terms are defined by dimensions:

$$\mathcal{L}_{\text{kin}} = \frac{N_c}{2}\text{tr}\left(-\sum_{j=1}^{M(M-1)/2} \bar{\phi}_i \left(\square^{D/2-\Delta_{\phi_i}}\right) \phi_i\right)$$

Alfimov, V.K., Olivucci '22

Generalized Fishnet CFT: Interactions

Construct the vertices: start from the largest one and consequtively replace pairs of fields by the "dual" fields (according to star-triangle):

$XY \to w$

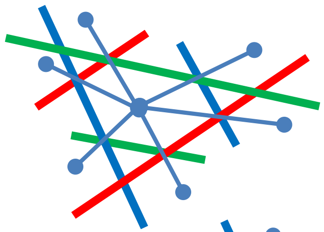
$YZ \to u$

$Z\bar{X} \to v$

General FCFT(3) has 18 vertices:

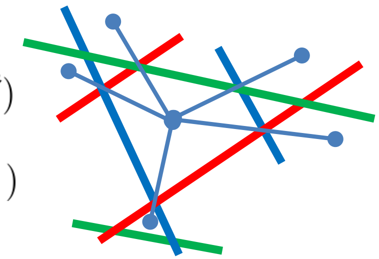
Each vertex has its independent coupling ξ_j

1 sextic $\quad \mathrm{tr}\,(XYZ\bar{X}\bar{Y}\bar{Z})$



6 quintic $\quad \mathrm{tr}\,(w\,Z\bar{X}\bar{Y}\bar{Z}) \quad \mathrm{tr}\,(X\,u\,\bar{X}\bar{Y}\bar{Z}) \quad \mathrm{tr}\,(XY\,v\,\bar{Y}\bar{Z})$

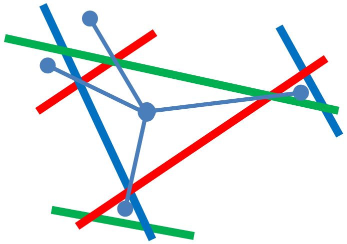
$\mathrm{tr}\,(XYZ\,\bar{w}\,\bar{Z}) \quad \mathrm{tr}\,(XYZ\bar{X}\,\bar{u}\,) \quad \mathrm{tr}\,(YZ\bar{X}\bar{Y}\,\bar{v}\,)$



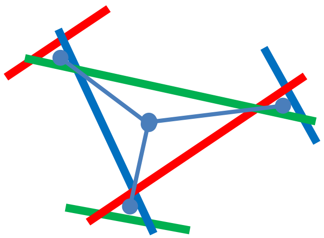
9 quartic $\quad \mathrm{tr}\,(w\,v\,\bar{Y}\bar{Z}) \quad \mathrm{tr}\,(w\,Z\bar{X}\,\bar{u}) \quad \mathrm{tr}\,(XY\,v\,\bar{u})$

$\mathrm{tr}\,(u\,\bar{X}\bar{Y}\,\bar{v}) \quad \mathrm{tr}\,(XY\,v\,\bar{u}) \quad \mathrm{tr}\,(YZ\,\bar{w}\bar{v})$

$\mathrm{tr}\,(Y\,v\,\bar{Y}\,\bar{v}) \quad \mathrm{tr}\,(w\,Z\,\bar{w}\,\bar{Z}) \quad \mathrm{tr}\,(u\,Z\bar{X}\,\bar{u}\,Z)$



2 cubic $\quad \mathrm{tr}\,(v\,\bar{w}\,\bar{u}) \quad \mathrm{tr}\,(w\,v\,\bar{u})$

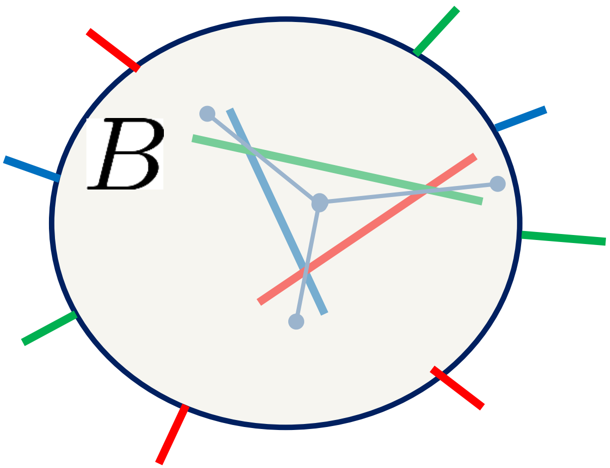


There are also double-trace terms…

V.K., Olivucci '22

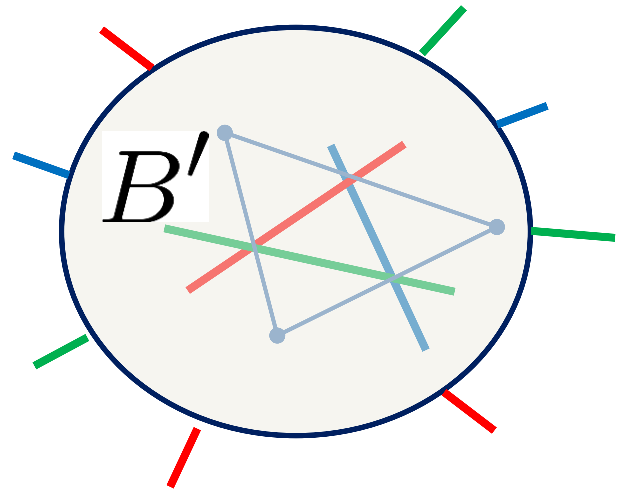
Summing Feynman graphs for Generalized Fishnet CFT

All Feynman graphs for a given planar correlator are connected by star-triangle relations: the "neighbors" differ by explicit factor $\mathcal{V}(G \to G')$

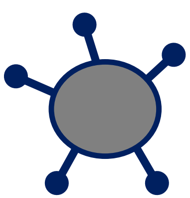


$$\mathcal{V}(B \to B') = V(a,b,c)$$

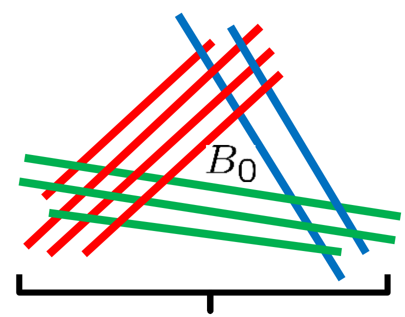
$$\mathcal{V}(B' \to B) = \frac{1}{V(a,b,c)}$$



Correlator is the sum of all such graphs. Any of these graphs is given by a single "base" graph times the product of factors due to star-triangle transformations from B0 to B:



$$= \sum_{B \in B_0} \left(\prod_{I \in B^*} \xi_I \right) \mathcal{V}(B_0 \to B_1)\, \mathcal{V}(B_1 \to B_2) \ldots \mathcal{V}(B_n \to B) \times$$



a single non-trivial graph to compute

Yangian symmetry!

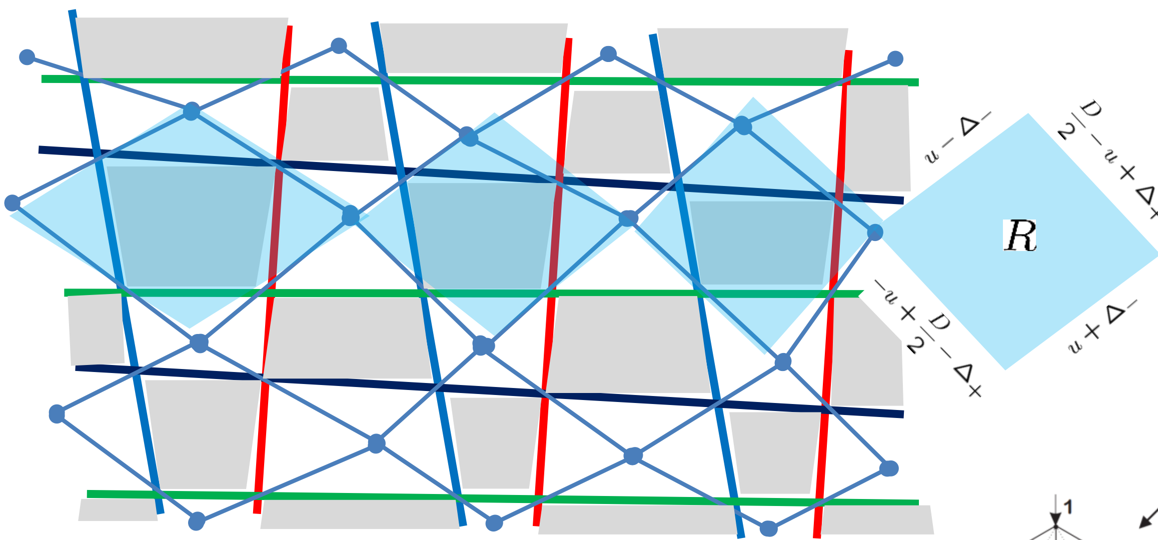
V.K., Levkovich-Maslyuk, Mishnyakov (in progress)

Alfimov, V.K., Fdrrando, Olivucci, to appear

Checkerboard FCFT$^{(4)}$

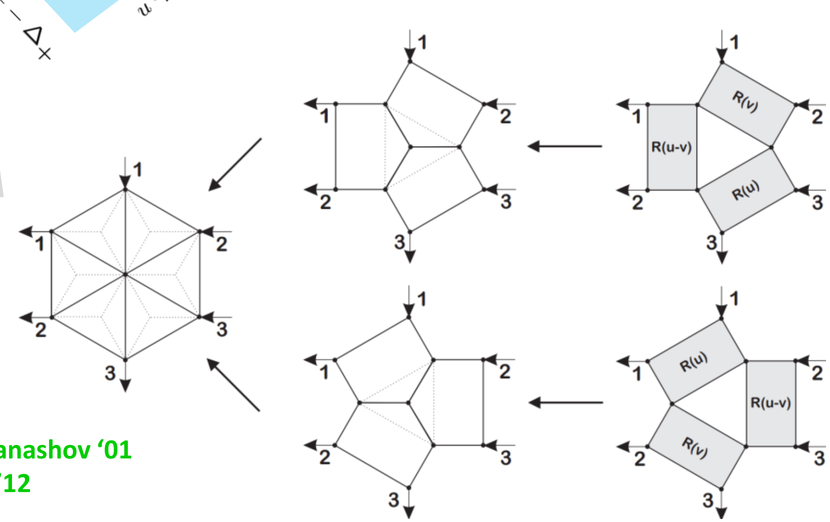
$$\mathcal{L}_D^{(CD)}\,|_{w_4=0} = N_c \mathrm{tr}\left[-\sum_{j=1}^{4} \bar{X}_j \Box^{w_j} X_j + \xi_1^2\, \bar{X}_1 \bar{X}_2 X_3 X_4 + \xi_2^2\, X_1 X_2 \bar{X}_3 \bar{X}_4\right], \qquad \sum_{j=1}^{4} w_j = D$$

Loom FCFT with 4 slopes but only two interaction terms turned on



Transfer-matrix

Yang-Baxter relation:



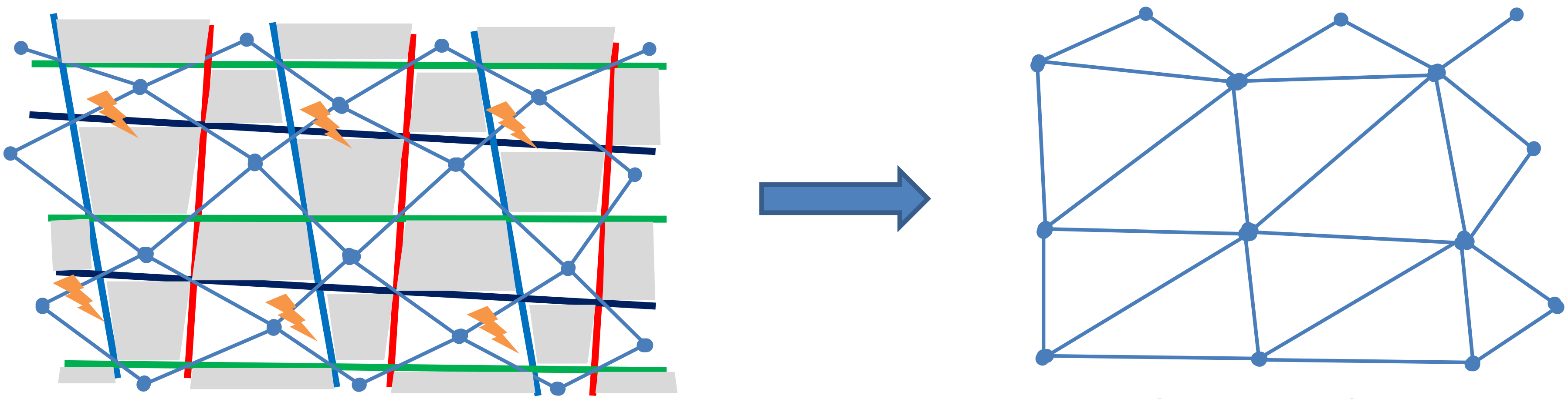
Derkachov, Korchemsky, Manashov ‘01
Chicherin, Derkachev, Isaev’12

ABJM from FCFT$^{(4)}$

- Reduction to FCFT$^{(3)}$: take $w_4 = 0$

$$\mathcal{L}_D^{(CD)}\,|_{w_4=0} = N_c \mathrm{tr}\left[-\sum_{j=1}^{3} \bar{X}_j \Box^{w_j} X_j - \bar{X}_4 X_4 + \xi_1^2\, \bar{X}_1 \bar{X}_2 X_3 X_4 + \xi_2^2\, X_1 X_2 \bar{X}_3 \bar{X}_4\right]$$

$$\to\ N_c \mathrm{tr}\left[-\sum_{j=1}^{3} \bar{X}_j \Box^{w_j} X_j + (\xi_1 \xi_2)^2\, \bar{X}_1 \bar{X}_2 X_3 X_1 X_2 \bar{X}_3\right]$$



- ABJM FCFT$^{(3)}$ emerges in particular case $D = 3\,, \quad w_1 = w_2 = w_3 = 1$

Alfimov, V.K., Ferrando, Olivucci, to appear

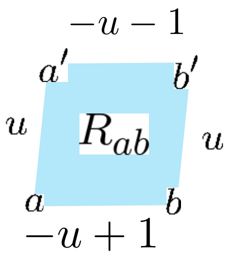
Particular case: “BFKL” fishnet CFT

- Fix the 2D Checkerboard parameters (angles of the loom) as follows

$$\mathcal{L}^{(BFKL)} = N_c \mathrm{Tr}\left[\bar{Z}_1(-\bar{\partial}\partial)^{u+2} Z_1 + \bar{Z}_2(-\bar{\partial}\partial)^{-u} Z_2 + \bar{Z}_3(-\bar{\partial}\partial)^{u} Z_3 + \bar{Z}_4(-\bar{\partial}\partial)^{-u} Z_4 + \right.$$
$$\left. + 4\pi\, \xi_1^2 \bar{Z}_1 \bar{Z}_2 Z_3 Z_4 + 4\pi\, \xi_2^2 Z_1 Z_2 \bar{Z}_3 \bar{Z}_4 \right]$$

$u \to 0$

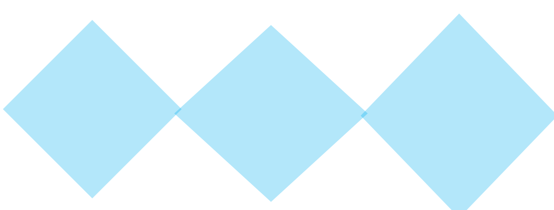
- Typical Feynman graphs constructed from R-matrices:



$$\hat{R}_{ab}^{BFKL} = (x_{ab}^2)^{u+1} (\hat{p}_b^2)^u (\hat{p}_a^2)^u (x_{ab}^2)^{u-1} = 1 + u\, \hat{h}_{ab}^{BFKL} + \mathcal{O}(u^2)$$

Chicherin, Derkachov, Isaev ‘03

$$\hat{h}_{ab}^{BFKL} = (p_a^2)^{-1} \log(x_{ab}^2)(p_a^2) + (p_b^2)^{-1} \log(x_{ab}^2)(p_b^2) + \log(p_a^2 p_b^2)$$



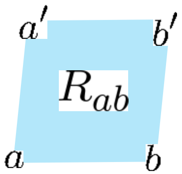
$$\hat{H}^{BFKL} = \sum_{a=1}^{L} \hat{h}_{a,a+1}^{BFKL}\,, \qquad (\hat{h}_{L,L+1} \equiv \hat{h}_{L,1})$$

Lipatov spin chain for interacting reggeized gluons

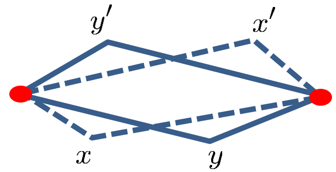
4-point correlator for Checkerboard FCFT(4)

- SO(2,D) spin chain of length=2. Solvable via only conformal symmetry!

R-matrix: product of 4 propagators

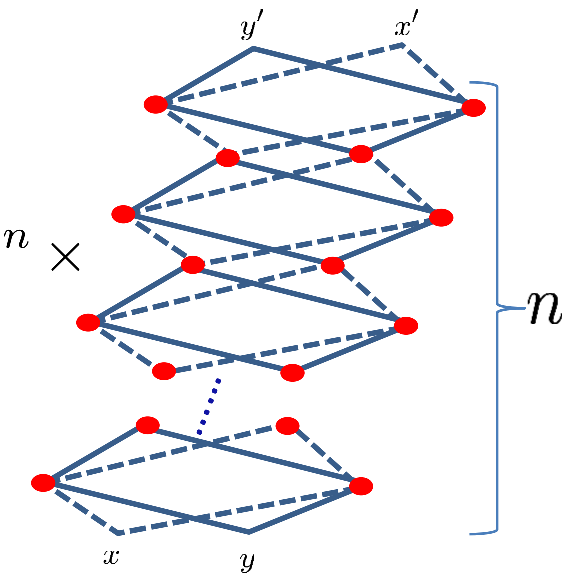


2-spin chain (graph-building operator)



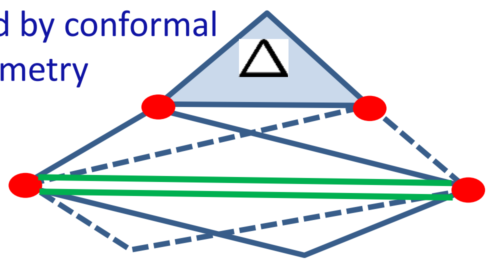
4-point correlator

$$\sum_n (\xi_1 \xi_2)^{2n} \times$$

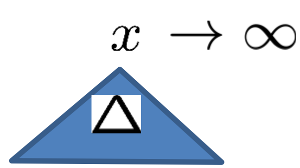


Diagonalization of Bethe-Salpeter operator

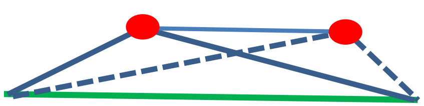
fixed by conformal symmetry



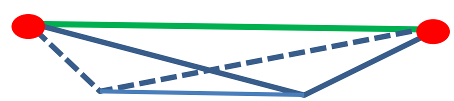
$$= h(\Delta)$$



Multiplying and devinding by the same (green) propagator we get two “kite” integrals:



$$= h_1(\Delta)$$



$$= h_2(\Delta)$$

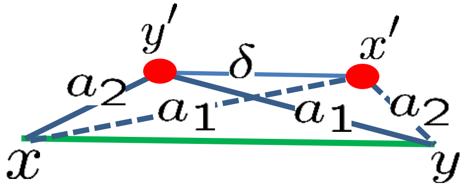
Spectrum of operators in OPE is fixed by poles:

$$h(\Delta) \equiv h_1(\Delta)\, h_2(\Delta) = \frac{1}{\xi_1^2 \xi_2^2}$$

Alfimov, V.K., Ferrando, Olivucci, to appear

General 4p correlator and particular cases

- Kite integral computed as double sums of ratios of Gamma functions



$$= \int\int d^D x' d^D y' (|x-x'||y-y'|)^{-2a_1} (|y-x'||x-y'|)^{-2a_1} |x-y'|^{-2\delta}$$
$$= C\,|x-y|^{2(D-\delta-2a_1-2a_2)}$$

Grozin
Derkachev et al'21

$$C = \pi^D \frac{\Gamma\left(\frac{D}{2}-1\right)}{\Gamma(D-2)} \frac{\mathbf{a}_0(a_1)\mathbf{a}_0(a_2)}{\mathbf{a}_0(2a_1+2a_2+\delta-D)} (I_1+I_2+I_3)\,, \quad \mathbf{a}_0(a) = \frac{\Gamma\left(\frac{D}{2}-a\right)}{\Gamma(a)}$$

$$M_n = \frac{1}{n!}\left(n+\frac{D}{2}-1\right)\Gamma(n+D-2)$$

$$I_1 = \sum_{n=0}^{+\infty}(-1)^n M_n \sum_{k=0}^{+\infty} \frac{(-1)^k}{k!} \frac{\Gamma(a_2+n+k)}{\Gamma\left(\frac{D}{2}-a_2-k\right)} \frac{\Gamma\left(\frac{D}{2}+n-\delta+k\right)}{\Gamma(\delta-k)} \frac{\Gamma\left(a_1+\delta-\frac{D}{2}-k\right)}{\Gamma(D+n-a_1-\delta+k)} \times$$
$$\times \frac{\Gamma(D+n-a_1-a_2-\delta+k)}{\Gamma\left(a_1+a_2+\delta-\frac{D}{2}-k\right)} \frac{\Gamma\left(\frac{D}{2}-a_1-a_2-k\right)}{\Gamma(n+a_1+a_2+k)} \frac{1}{\Gamma\left(\frac{D}{2}+n+k\right)}\,, \qquad (4.19)$$

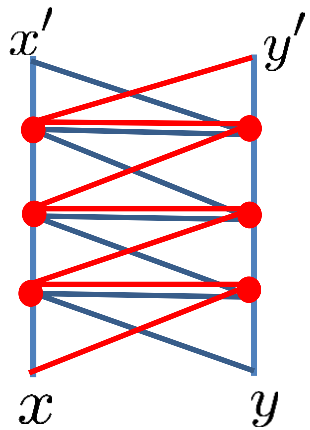
$$I_2 = \sum_{n=0}^{+\infty}(-1)^n M_n \sum_{k=0}^{+\infty} \frac{(-1)^k}{k!} \frac{\Gamma\left(n-\frac{D}{2}+a_1+a_2+\delta+k\right)}{\Gamma(D-a_1-a_2-\delta-k)} \frac{\Gamma\left(\frac{D}{2}-a_1-\delta-k\right)}{\Gamma(n+a_1+\delta+k)} \frac{\Gamma(n+a_1+k)}{\Gamma\left(\frac{D}{2}-a_1-k\right)} \times$$
$$\times \frac{\Gamma\left(\frac{D}{2}+n-a_2+k\right)}{\Gamma(a_2-k)} \frac{\Gamma(D-2a_1-a_2-\delta-k)}{\Gamma(n-D+2a_1+a_2+\delta+k)} \frac{1}{\Gamma\left(\frac{D}{2}+n+k\right)}\,, \qquad (4.20)$$

$$I_3 = \sum_{n=0}^{+\infty}(-1)^n M_n \sum_{k=0}^{+\infty} \frac{(-1)^k}{k!} \frac{\Gamma\left(\frac{D}{2}+n-a_1+k\right)}{\Gamma(a_1-k)} \frac{\Gamma\left(a_1+a_2-\frac{D}{2}-k\right)}{\Gamma(D+n-a_1-a_2+k)} \frac{\Gamma(D+n-a_1-a_2-\delta+k)}{\Gamma(a_1+a_2+\delta-k)} \times$$
$$\times \frac{\Gamma(2a_1+a_2+\delta-D-k)}{\Gamma\left(\frac{3D}{2}+n-2a_1-a_2-\delta+k\right)} \frac{\Gamma\left(\frac{3D}{2}+n-2a_1-2a_2-\delta+k\right)}{\Gamma(2a_1+2a_2+\delta-D-k)} \frac{1}{\Gamma\left(\frac{D}{2}+n+k\right)}\,. \qquad (4.21)$$

Alfimov, V.K., Ferrando, Olivucci, to appear

Particular case: "ABJM" Fishnet CFT

- ABJM D=3: 1st and 2nd double-sums are hypergeometric functions; 3rd double-sum is infinite sum of hypergeometric functions.
  The perturbation theory up to 3 orders for the anomalous dimension:



$$-\frac{a}{\gamma^2}+\frac{b}{\gamma}+c+\mathcal{O}(\gamma)=\frac{1}{\zeta}$$

$$\zeta=(\xi_1\xi_2)^2$$

$$a=\frac{1}{2^{12}}\left(\frac{2}{\pi^{\frac{7}{2}}}+\frac{5\sqrt{2}\log(\sqrt{2}+1)}{\pi^5}-\frac{2}{\pi^5}\right),$$

$$b=\frac{1}{2^{10}}\left(\frac{G}{\pi^{\frac{9}{2}}}+\frac{\log 2}{\pi^{\frac{7}{2}}}-\frac{5\mathrm{Li}_2\left(\frac{1}{\sqrt{2}}\right)}{2\sqrt{2}\pi^5}+\frac{5\log 2\log(\sqrt{2}+1)}{\sqrt{2}\pi^5}-\frac{5\log^2 2}{16\sqrt{2}\pi^5}+\frac{5}{96\sqrt{2}\pi^5}-\right.$$
$$\left.-\frac{\log(\sqrt{2}+1)}{2\sqrt{2}\pi^5}-\frac{\log 2}{\pi^5}\right),$$

$$c=\frac{1}{2^{12}}\left(\frac{2i\left(\mathrm{Li}_3\left(\frac{1+i}{2}\right)-\mathrm{Li}_3\left(\frac{1-i}{2}\right)\right)}{\pi^{\frac{9}{2}}}-\frac{6G\log 2}{\pi^{\frac{9}{2}}}-\frac{33\log^2 2}{8\pi^{\frac{7}{2}}}+\frac{1}{96\pi^{\frac{3}{2}}}+\frac{35\zeta_3}{32\sqrt{2}\pi^5}-\right.$$
$$-\frac{5\sqrt{2}\mathrm{Li}_3\left(\frac{1}{\sqrt{2}}\right)}{\pi^5}+\frac{10\sqrt{2}\log 2\mathrm{Li}_2\left(\frac{1}{\sqrt{2}}\right)}{\pi^5}+\frac{65\log^3 2}{24\sqrt{2}\pi^5}-\frac{10\sqrt{2}\log^2 2\log(\sqrt{2}+1)}{\pi^5}+$$
$$+\frac{5\log(\sqrt{2}+1)}{6\sqrt{2}\pi^3}-\frac{25\log 2}{48\sqrt{2}\pi^3}+\frac{5}{18\pi^2}-\frac{\sqrt{2}\mathrm{Li}_2\left(\frac{1}{\sqrt{2}}\right)}{\pi^5}+\frac{2\sqrt{2}\log 2\log(\sqrt{2}+1)}{\pi^5}+$$
$$\left.+\frac{(16\sqrt{2}-1)\log^2 2}{\pi^5}-\frac{44\log 2}{\pi^4}+\frac{(4\sqrt{2}-1)}{24\sqrt{2}\pi^3}-\frac{4}{9\pi^4}\right).$$

- Inverting the $\varkappa$-expansion we find the perturbative series:

$$\gamma=\pm i\sqrt{a\zeta}\left(1-\frac{b^2-4ac}{8a}\zeta\right)+\frac{b}{2}\zeta+\mathcal{O}\left(\zeta^2\right)$$

Alfimov, V.K., Ferrando, Olivucci, to appear

Particular case: “BFKL” Fishnet CFT

- For the BFKL-like choice of the parameters of Checkerboard FCFT, in the limit *u→0,* we reproduce the analog of BFKL pomeron spectrum

$$h(\Delta) = 1 + 4(2\ln 2 + \psi(\Delta) + \psi(1-\Delta))u + \mathcal{O}(u^2)$$

- The equation for anomalous dimensions takes the form

$$\eta = 4(2\psi(1) - \psi(\Delta) - \psi(1-\Delta))$$

rescaled coupling constant

$$(\xi_1\xi_2)^2 = \frac{u^4(1+u\eta)}{\pi^4}$$

Comments and Prospects

- We constructed Loom Fishnet CFTs based on Integrable planar graphs (dual to Baxter lattices)
- Fermions and arbitrary spins can be included into Generalized Fishnet CFT$^{(M)}$
- Certain double trace counter-terms can occur in the action of Fishnet CFT$^{(M)}$. They are important to cancel divergencies at cylindric topology of graphs
- Computation of scaling dimensions, OPE coefficients and n-point functions
- Generalized fishnet amplitudes. Yangian symmetry of the loom FCFTs
- Supersymmetrize the loom?
- Can we lift some of these Fishnet CFT$^{(M)}$ to the full-flagged susy gauge theory?
- Condensed matter and quantum physics applications for Fishnet CFT?


Thank you!
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