PERIMETER INSTITUTE

SRS
SwissMAP Research Station in Les Diablerets

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TRIANGULATION OF FISHNET DIAGRAMS

An application of spin-chain integrability to perturbative CFT in 4d

Les Diablerets - February 9th 2023

Integrability in Condensed Matter Physics and QFT

BASED ON :

- S. Derkachov, E.O. [2103.01940],[2007.15049],[1912.07588].
- E.O. [2107.13035],[23xx.xxxx].
- F. Aprile, E.O. [23xx.xxxx]

AND ALSO ON :

- S. Derkachov, V. Kazakov, E.O. [1811.10623].
- S. Derkachov, G. Ferrando, E.O.[2108.12620].
- V. Kazakov, E.O. [2212.09732]

PLAN OF THE TALK

- *Integrable non-compact spin chain SO(1,5)*
- *Separation of Variables in Fishnet diagrams*
- *General Fishnet on the disk:*
  - *Cutting*
  - *Overlapping*
  - *Gluing*
- *Light-cone limit: abelian truncation.*
- *Beyond Fishnet Theory, beyond the disk, and beyond.*

FISHNET THEORY: I

- Simple interaction: quartic chiral vertex.

$$\mathscr{L} = \mathbf{Tr}\left[\partial^\mu X \partial_\mu \bar{X} + \partial^\mu Z \partial_\mu \bar{Z} + \xi^2 X Z \bar{X} \bar{Z}\right]$$

[Gurdogan, Kazakov '15]

- X, Z matrix scalar fields in the adjoint rep. of $SU(N)$.

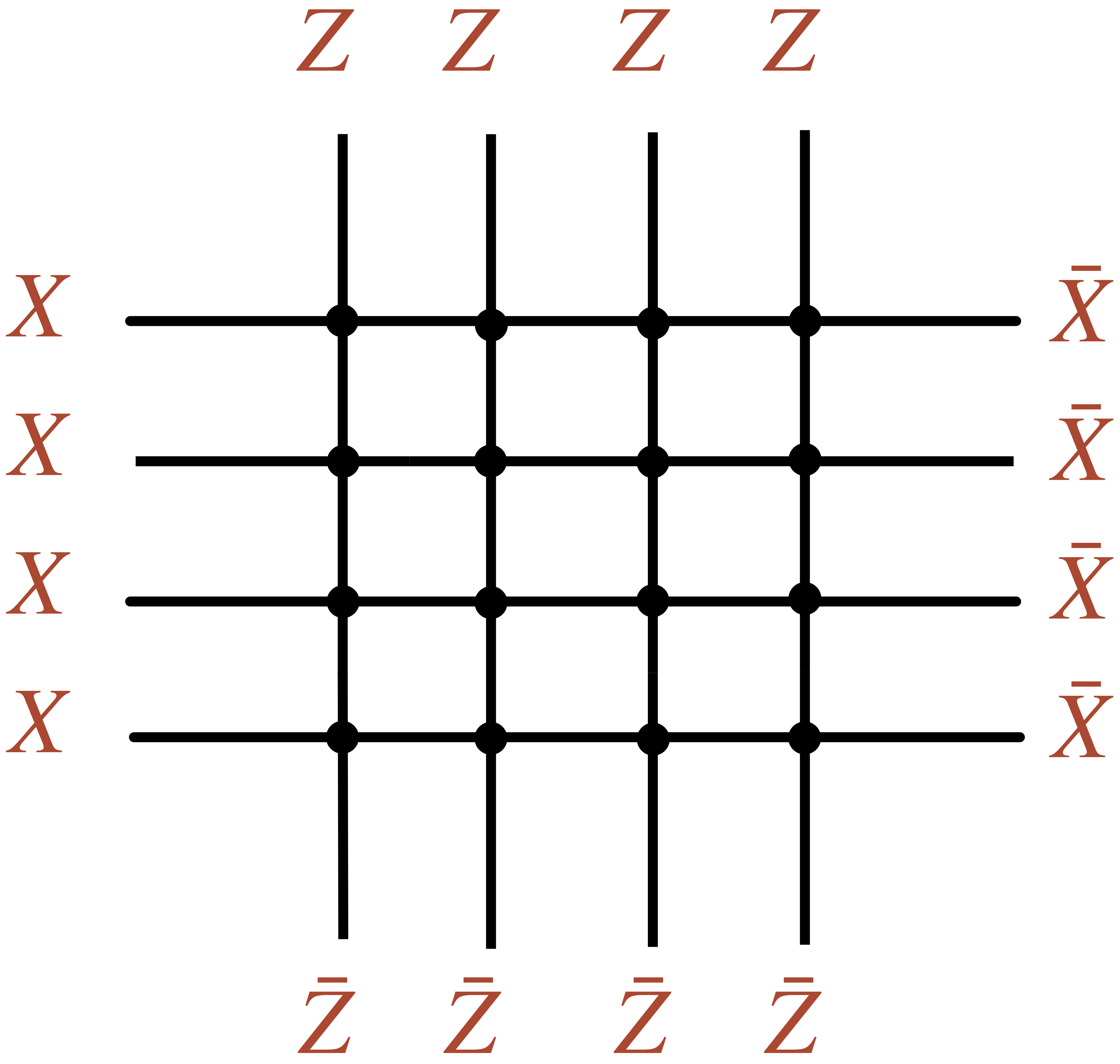
- Quartic vertex: chiral interaction $\mathbf{Tr}\left[XZ\bar{X}\bar{Z}\right] \neq \mathbf{Tr}\left[XZ\bar{X}\bar{Z}\right]^\dagger$

- Multicolor (planar) limit $N \gg 1$: no mass generation, no loop corrections to the coupling.

- Any Feynman integral in the theory: a portion of Fishnet square-lattice + b.c.

FISHNET FEYNMAN DIAGRAMS

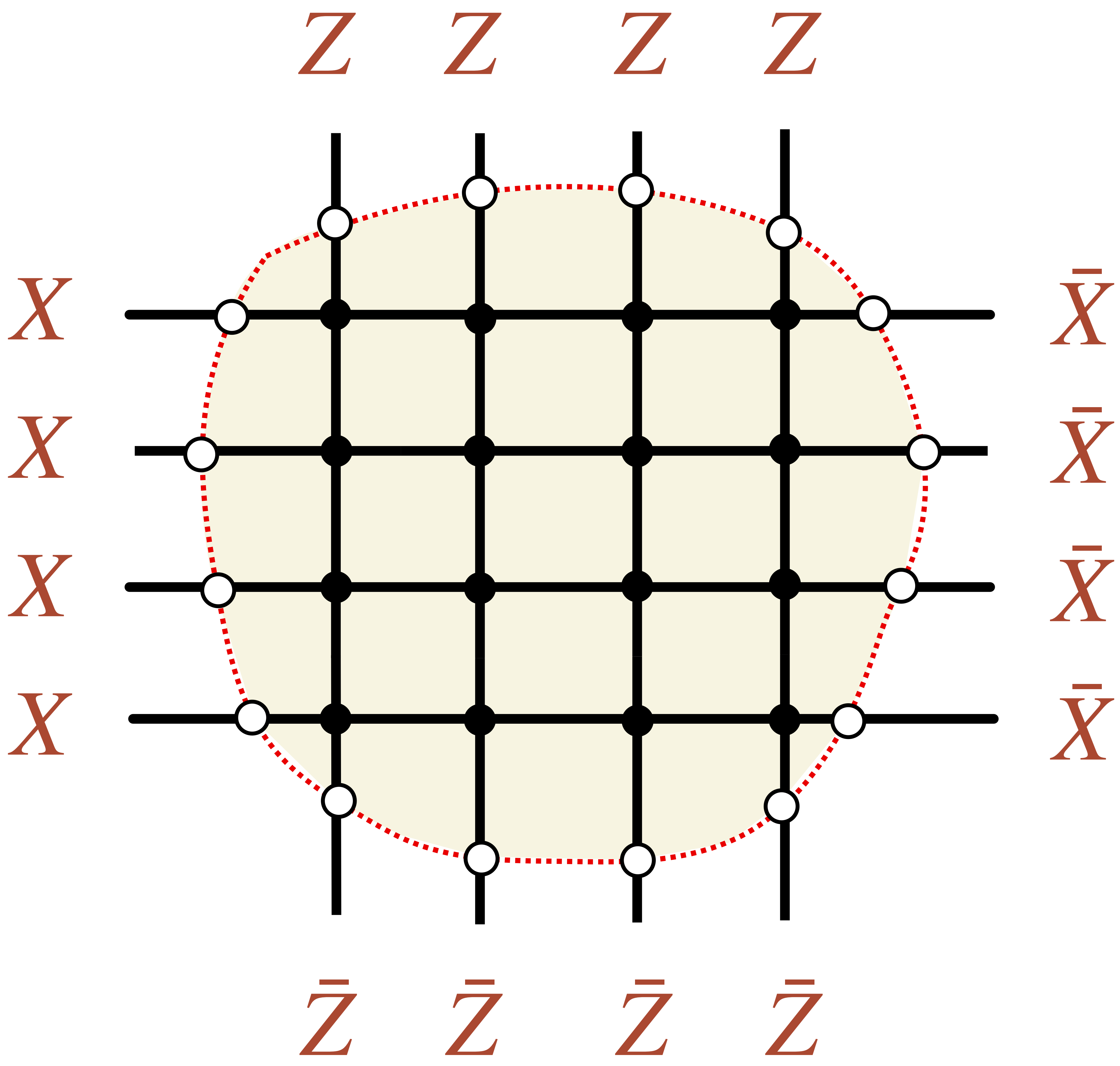
$$\mathrm{Tr}\left[XZ\bar{X}\bar{Z}\right]$$



 $= \dfrac{1}{(y_1 - y_2)^2}$

FISHNET FEYNMAN DIAGRAMS

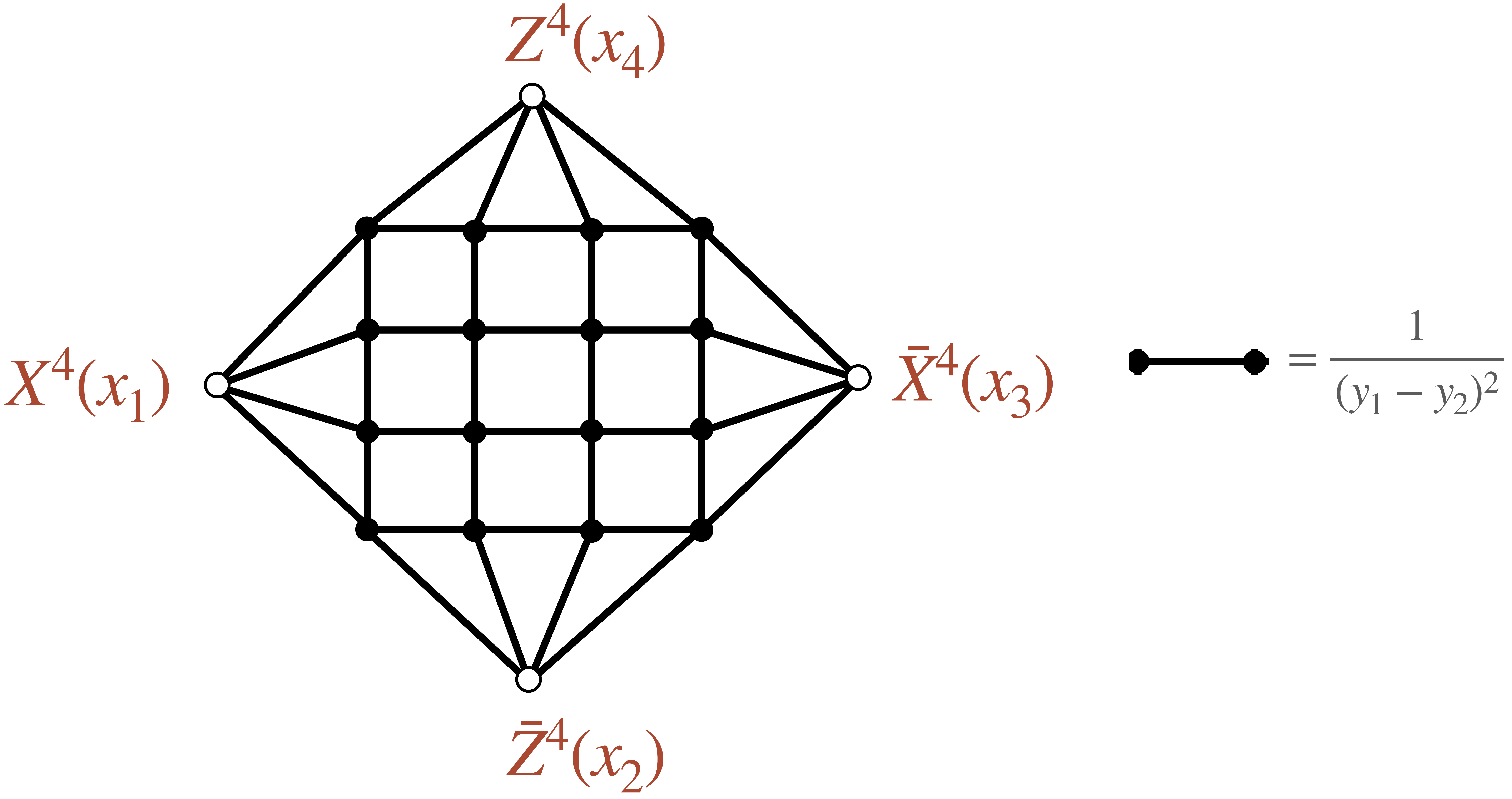
$\mathbf{Tr}\left[XZ\bar{X}\bar{Z}\right]$



 $= \dfrac{1}{(y_1 - y_2)^2}$

$\langle \mathbf{Tr}\left[X^4 Z^4 \bar{X}^4 \bar{Z}^4\right] \rangle$

FISHNET FEYNMAN DIAGRAMS



$$\langle \mathbf{Tr} \left[X^4(x_1) Z^4(x_2) \bar{X}^4(x_3) \bar{Z}^4(x_4) \right] \rangle$$

FISHNET FEYNMAN DIAGRAMS

$$I_{m,n} = \sum_{\mathbf{a}} \int \frac{d\mathbf{u}}{m!} \prod_{i=1}^{m} \frac{a_i z^{-iu_i + a_i/2} \bar{z}^{-iu_i - a_i/2}}{(u_i^2 + a_i^2/4)^{m+n}} \prod_{i<j}^{m} \Delta_{ij}$$

with $\Delta_{ij} = \Delta_{a_i a_j}(u_i, u_j)$

$$\Delta_{ab}(u,v) = \left[(u-v)^2 + \frac{(a-b)^2}{4}\right]\left[(u-v)^2 + \frac{(a+b)^2}{4}\right]$$

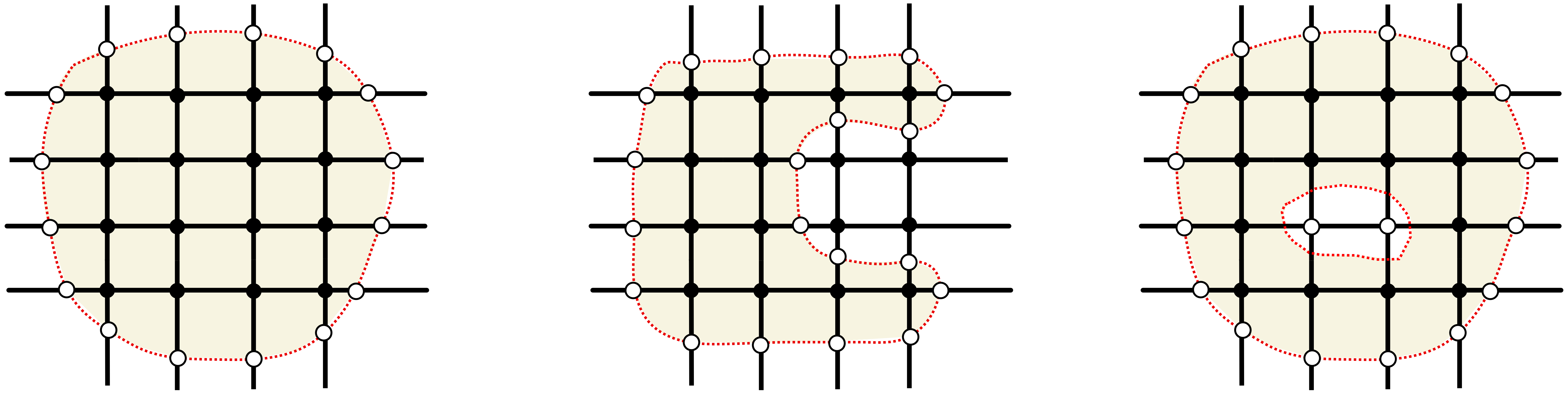
$$I_{m,n} = \frac{1}{\mathcal{N}} \det_{1 \leqslant i,j \leqslant m} (M_{i+j+n-m-1})$$

[Basso-Dixon '16]

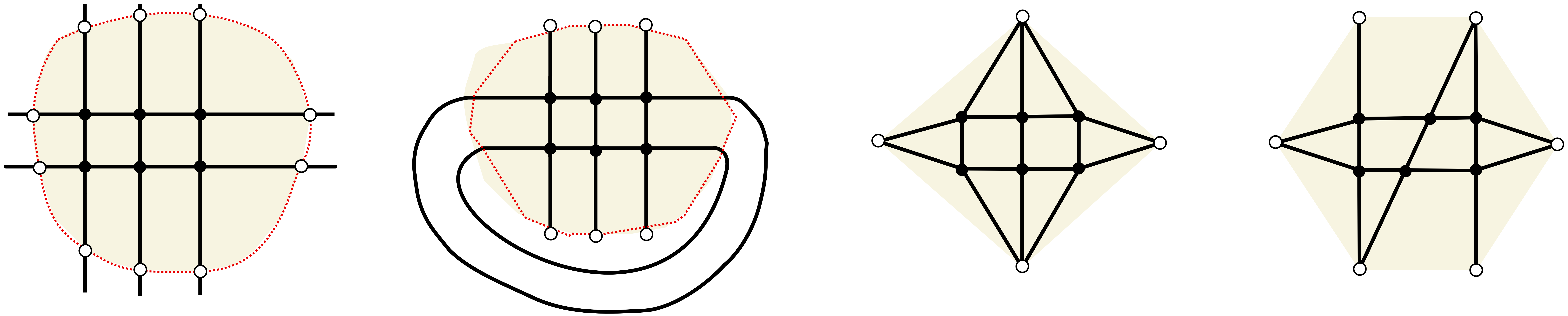
$$f_L(z,\bar{z}) = \sum_{j=L}^{2L} \frac{(L-1)!j!}{(j-L)!(2L-j)!} \frac{\mathrm{Li}_j(z) - \mathrm{Li}_j(\bar{z})}{z - \bar{z}} (-\log(z\bar{z}))^{2L-j}$$

Also: [S. Derkachov, E.O. '20]+[Basso,Dixon,Kosower,Krajenbrink,Zhong '21]

FISHNET THEORY: II



Yangian Symmetry with conformal group SO(1,5) *[Chicherin,Kazakov, Loebbert, Mueller,Zhong ’17]*



Boundary conditions: punctured sphere vs disk, UV divergent vs finite.

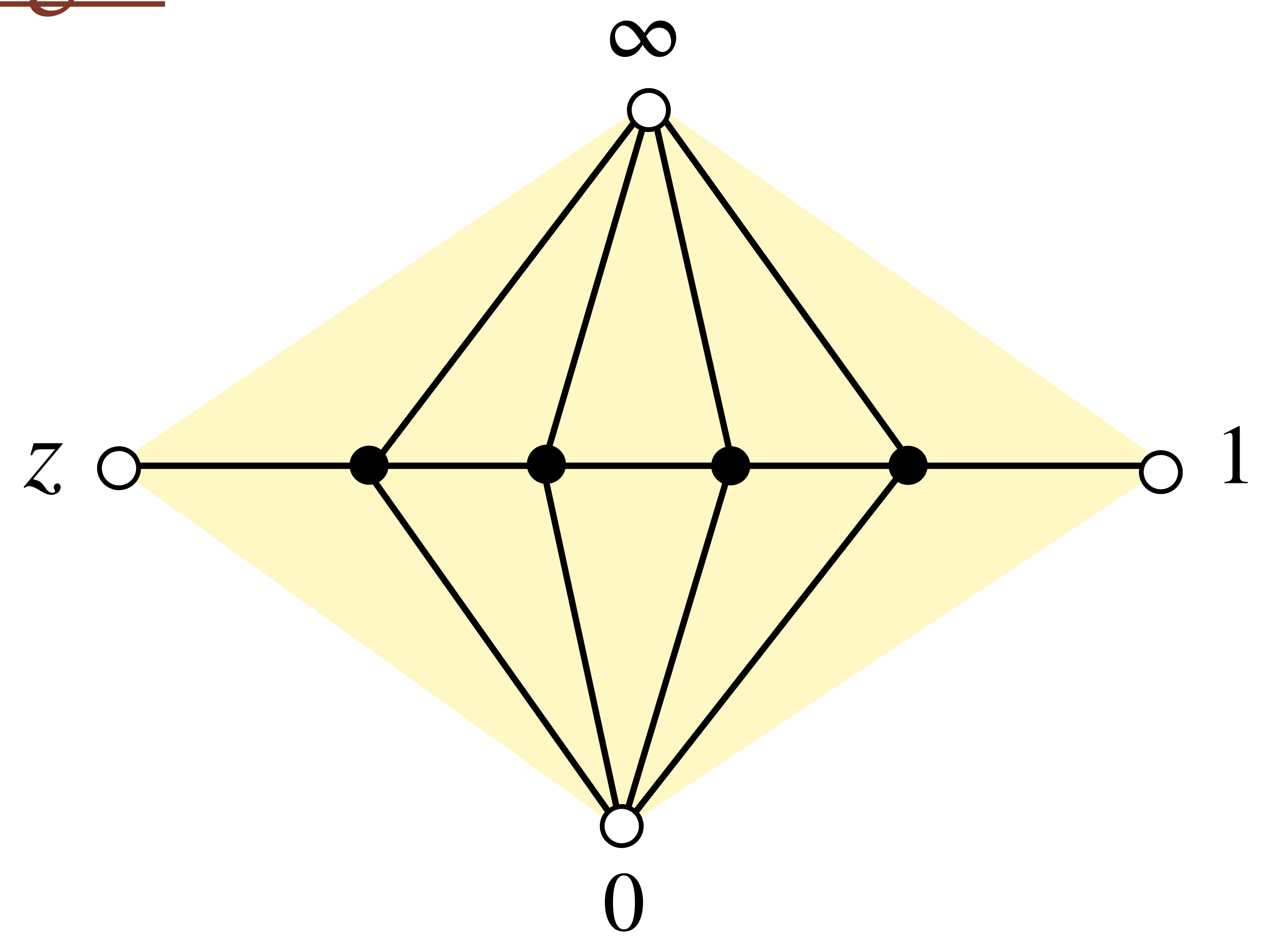
FISHNET THEORY: III

- Why is it worth studying?

  CFT in 4d, log-CFT (non-unitary), related to AdS/CFT correspondence, “tT”-symmetry, integrability of scaling dimension (QSC), related to BFKL hamiltonian, **spin-chain methods, exactly solvable Feynman integrals.**

- A toy-model for the toy-model (N=4 SYM): proving conjectures in the Fishnet limit.
- Exploration of the basis of functions of Feynman integrals
- In some dynamical regime of N=4 SYM, Fishnets are enough:

$$f_L(z,\bar{z}) = \sum_{j=L}^{2L} \frac{(L-1)!j!}{(j-L)!(2L-j)!} \frac{\text{Li}_j(z) - \text{Li}_j(\bar{z})}{z-\bar{z}} (-\log(z\bar{z}))^{2L-j}$$



N=4 SYM OCTAGON

- In some dynamical regime of N=4 SYM, Fishnets are enough:

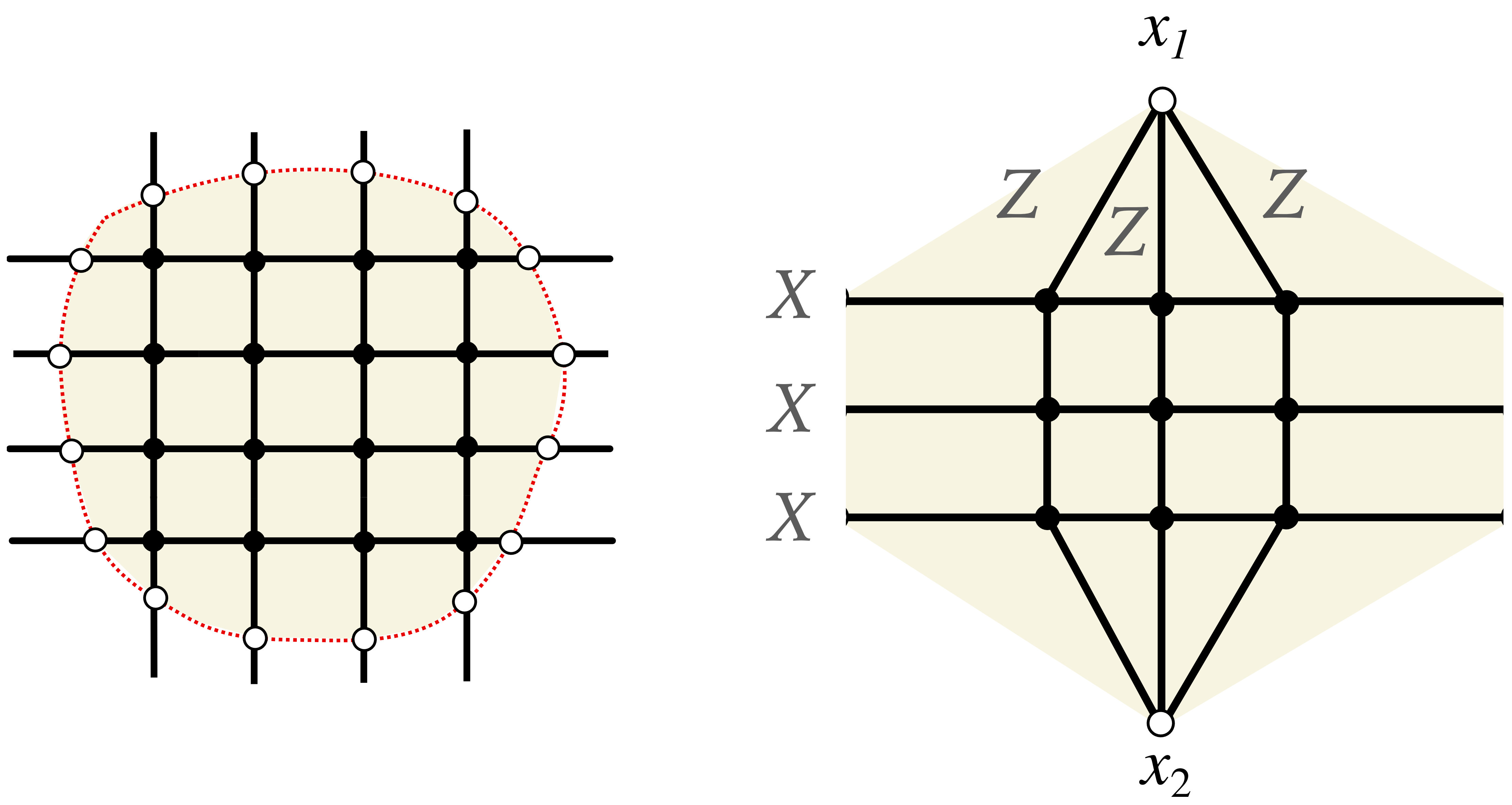
$$\mathbb{O}_l(z,\bar{z}) = \langle Tr(y_1\cdot\Phi)^{L_1}\, Tr(y_2\cdot\Phi)^{L_2}\, Tr(y_3\cdot\Phi)^{L_3}\, Tr(y_4\cdot\Phi)^{L_4}\rangle$$

[Coronado '18]

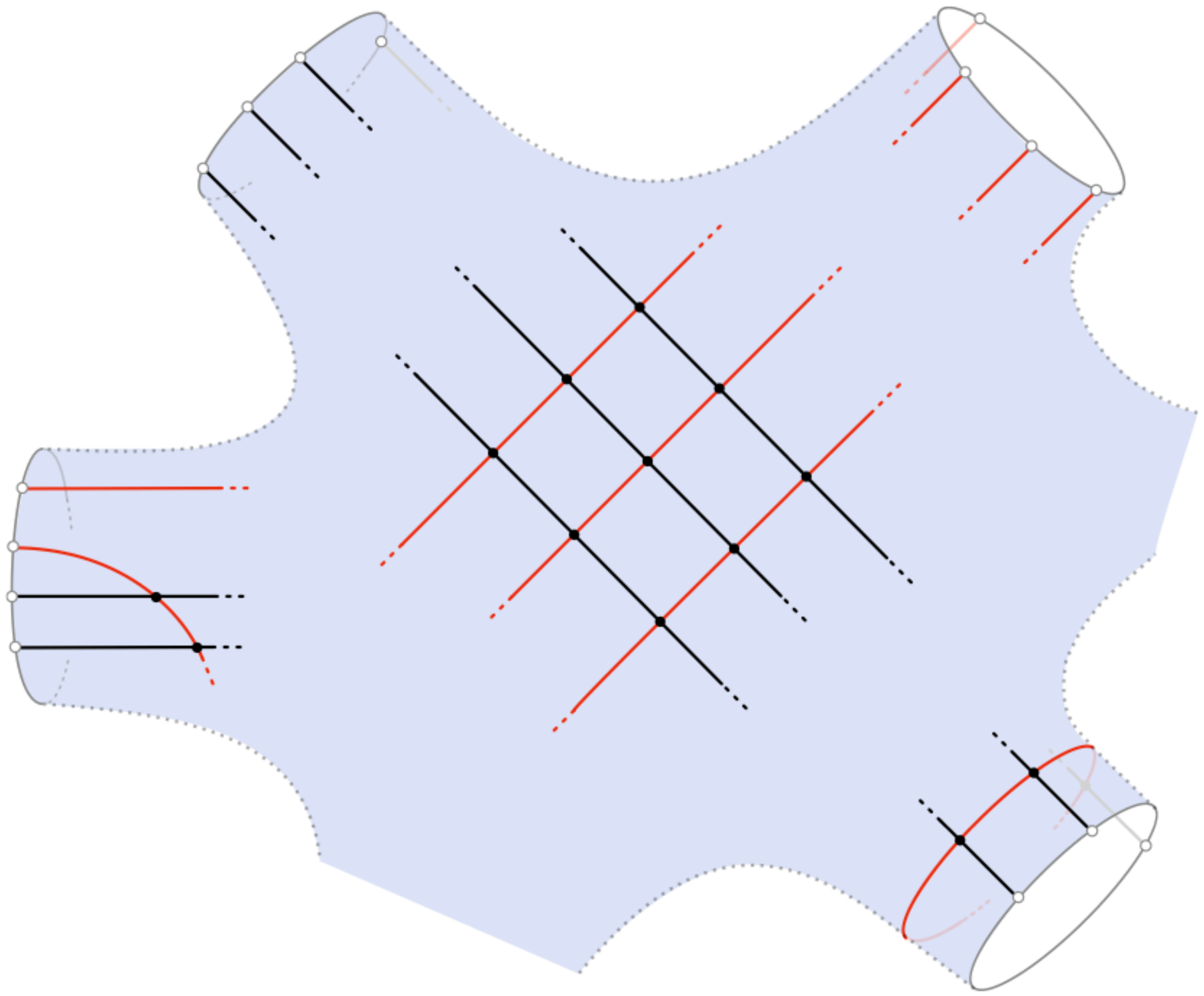
$$\mathbb{O}_l = 1+\sum_{n=1}^{\infty}(z+\bar{z}-z\bar{z})^n \sum_{m=n(n+l)}^{\infty} \lambda^m \sum_{\underline{k}} \frac{c^{(l)}_{k_1,\dots,k_n}}{\prod_{j=1}^{n} k_j!(k_j-1)!} \begin{vmatrix} f_{k_1} & f_{k_2-1} & \cdots & f_{k_n-n+1} \\ f_{k_1+1} & f_{k_2} & \cdots & f_{k_n-n+2} \\ \vdots & \vdots & \cdots & \vdots \\ f_{k_1+n-1} & f_{k_2+n-2} & \cdots & f_{k_n} \end{vmatrix}$$

FIELD THEORY

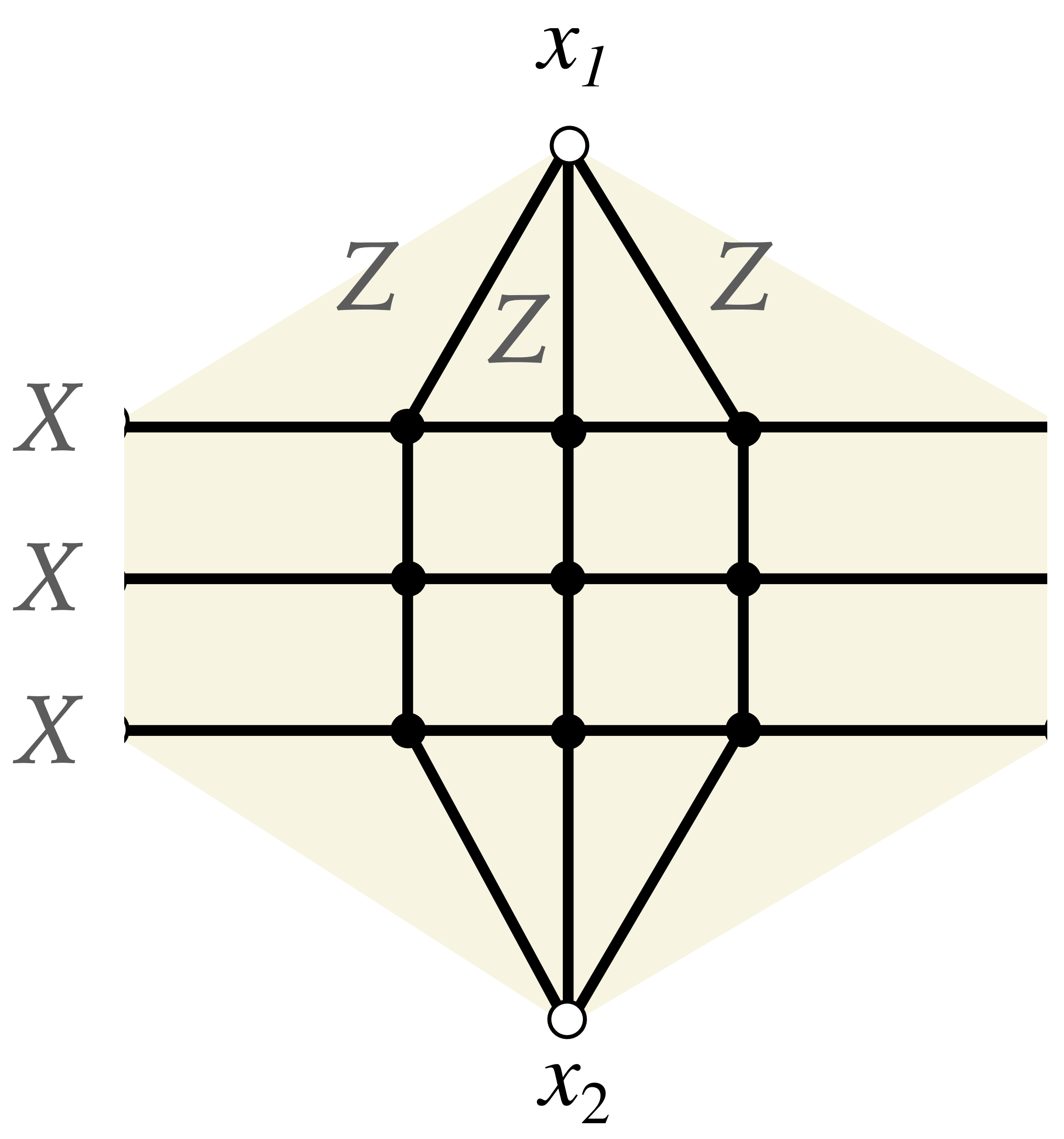
- Single-trace disk correlators
- Single-trace correlators $\langle \mathrm{Tr}\left[X^L Z^N \bar{X}^L \bar{Z}^N\right]\rangle$

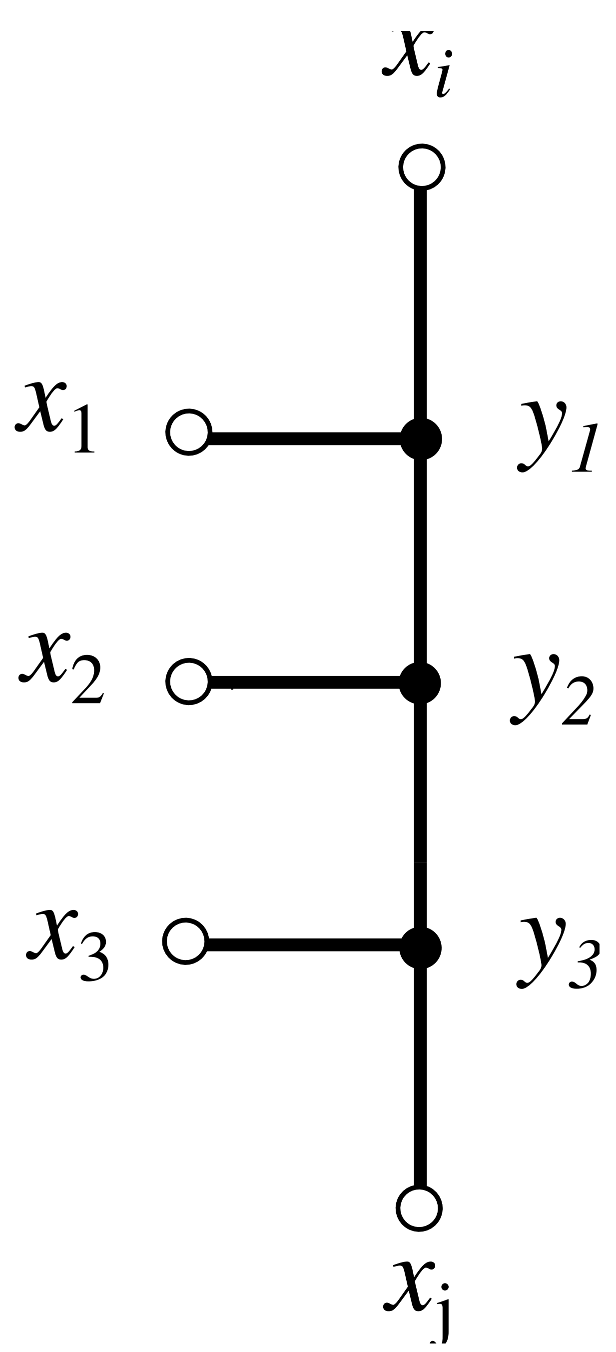


- Versus ``gauge-symmetric" version
  e.g.: $\langle \mathrm{Tr}\left[X^L\right] \mathrm{Tr}\left[Z^N\right] \mathrm{Tr}\left[\bar{X}^L\right] \mathrm{Tr}\left[\bar{Z}^N\right]\rangle$



SPIN CHAIN FROM FEYNMAN DIAGRAMS: I





- X, Z in the irreducible unitary representation $(\Delta, s, \dot{s}) = (1,0,0)$ (complementary series)

- There is a suitable spin-chain model!

$$\hat{B}_{ij}^{(L)}\, \Phi(x_1, \ldots, x_L) = \frac{1}{(4\pi^2)^{2L+2}} \int \left(\prod_{k=1}^{L} \frac{d^4 y_k}{(y_k - y_{k+1})^2 (x_k - y_k)^2} \right) \frac{\Phi(y_1, \ldots, y_L)}{(x_i - y_1)^2 (y_L - x_j)^2} .$$

SPIN CHAIN FROM FEYNMAN DIAGRAMS: II

Field representation

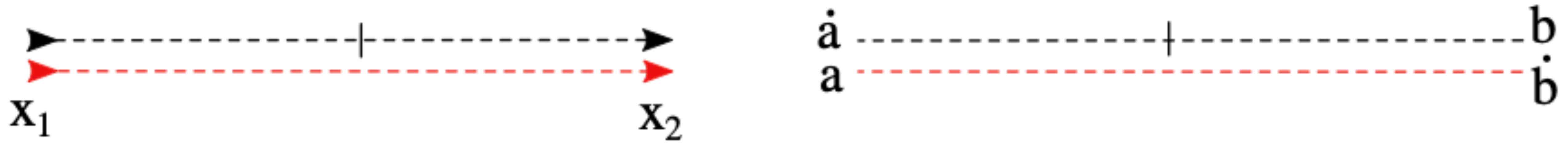
$$(\Delta, \ell, \dot{\ell})$$

$$\Delta = 2 + i\nu \,, \; \ell, \dot{\ell} \in \mathbb{N}$$

Bare propagator

$$\frac{[\mathbf{x}]^{\ell}[\overline{\mathbf{x}}]^{\dot{\ell}}}{(x^2)^{\Delta}} \longrightarrow \lambda(x)^{-2\Delta} \frac{[U\mathbf{x}V^{\dagger}]^{\ell}[V\overline{\mathbf{x}}U^{\dagger}]^{\dot{\ell}}}{(x^2)^{\Delta}}$$

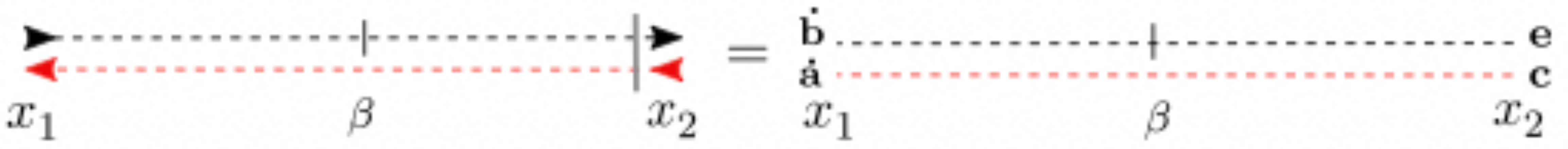
$$\mathbf{x} = \boldsymbol{\sigma}_{\mu} \frac{x^{\mu}}{(x^2)^{\frac{1}{2}}} \,, \quad \overline{\mathbf{x}} = \overline{\boldsymbol{\sigma}}_{\mu} \frac{x^{\mu}}{(x^2)^{\frac{1}{2}}}$$



Fused su(2) R-matrix

$$(\mathbf{R}_{mn})^{\mathbf{bd}}_{\mathbf{ac}}(u) = \mathbf{R}^{(b_1 \ldots b_m)(d_1 \ldots d_n)}_{(a_1 \ldots a_m)(c_1 \ldots c_n)}(u)$$

$$\frac{[\overline{\mathbf{x}}_{12}]^{\mathbf{e}}_{\dot{\mathbf{b}}} \mathbf{R}^{\mathbf{bc}}_{\mathbf{ea}}(u)[\mathbf{x}_{12}]^{\dot{\mathbf{a}}}_{\mathbf{c}}}{(x_{12})^{2\beta}} =$$



$$\mathbf{R}^{\mathbf{bc}}_{\mathbf{ea}}(u)$$

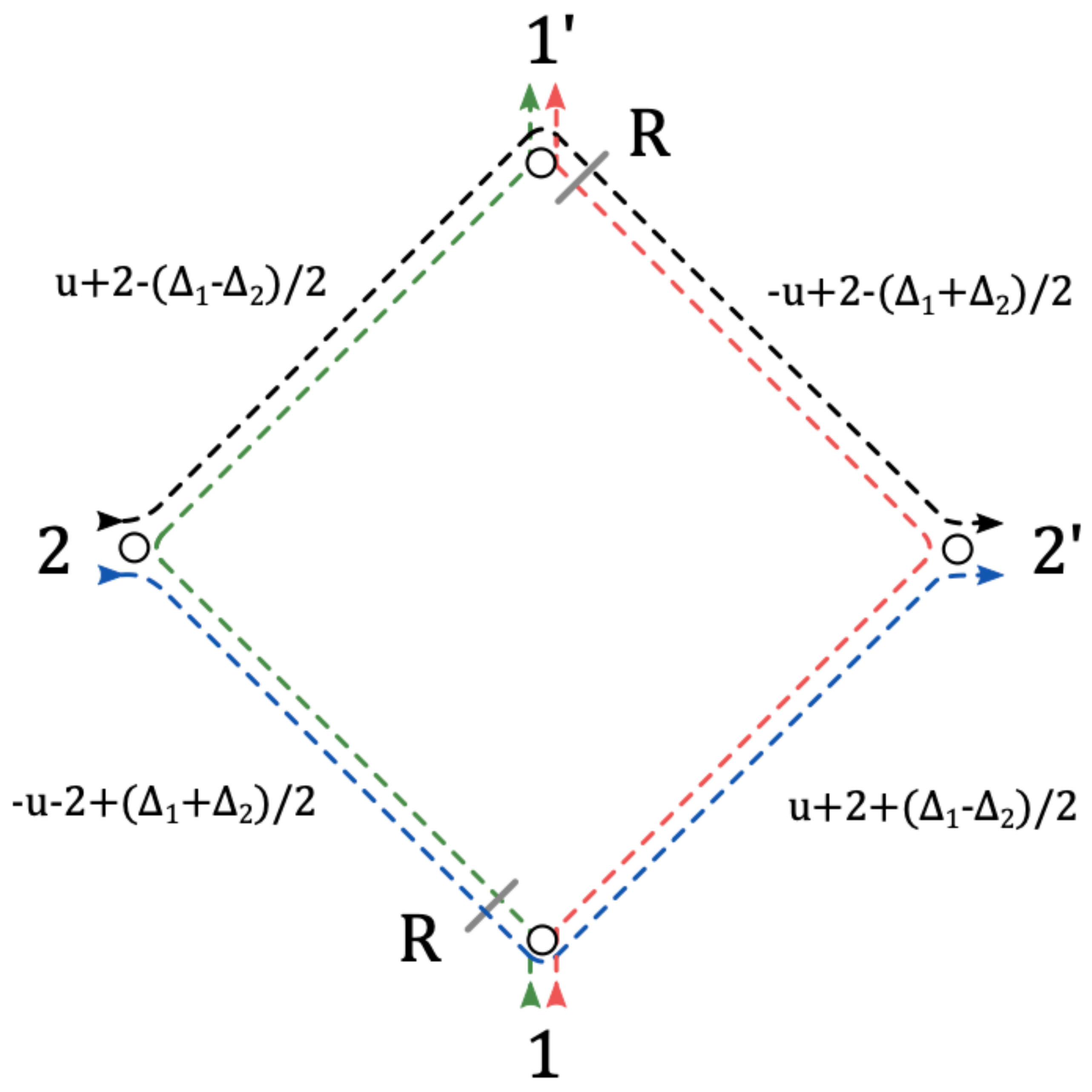
Ising type model “the edges”

SPIN CHAIN FROM FEYNMAN DIAGRAMS: III

$$\frac{[\mathbf{x}]^{\ell}[\overline{\mathbf{x}}]^{\dot{\ell}}}{(x^2)^{\Delta}}$$

$$(\mathbf{R}_{mn})^{\mathbf{bd}}_{\mathbf{ac}}(u)$$

- Infinite-dimensional R-matrix: solution of YBE by means of Star-Triangle Identity



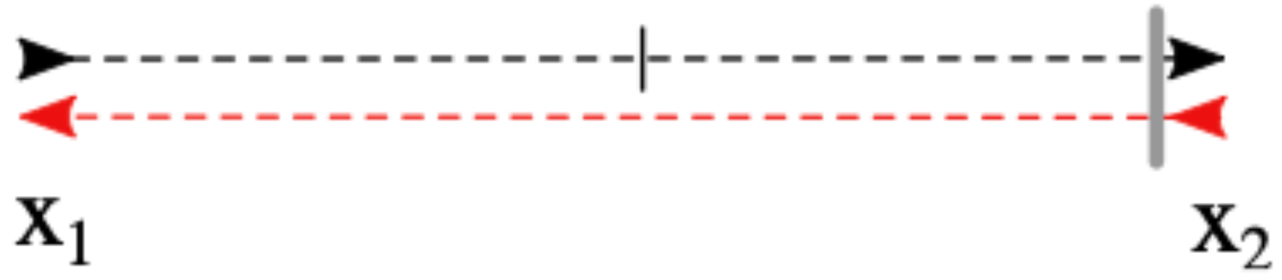
$$\mathcal{R}_{12}(u) = \mathbb{P}_{12}\frac{[\overline{\mathbf{x}}_{12}]^{\dot{\ell}_1}\mathbf{R}_{\dot{\ell}_1\ell_2}(u-\Delta_+)[\mathbf{x}_{12}]^{\ell_2}}{x_{12}^{2(-u+\Delta_+)}}\,\frac{[\overline{\mathbf{p}}_2]^{\ell_2}\mathbf{R}_{\ell_2\ell_1}(u+\Delta_-)[\mathbf{p}_2]^{\ell_1}}{\hat{p}_2^{2(-u-\Delta_-)}}\times$$
$$\times\,\frac{[\overline{\mathbf{p}}_1]^{\dot{\ell}_2}\mathbf{R}_{\dot{\ell}_2\dot{\ell}_1}(u-\Delta_-)[\mathbf{p}_1]^{\dot{\ell}_1}}{\hat{p}_1^{2(-u+\Delta_-)}}\,\frac{[\overline{\mathbf{x}}_{12}]^{\ell_1}\mathbf{R}_{\ell_1\dot{\ell}_2}(u+\Delta_+)[\mathbf{x}_{12}]^{\dot{\ell}_2}}{x_{12}^{2(-u-\Delta_+)}}\,,$$

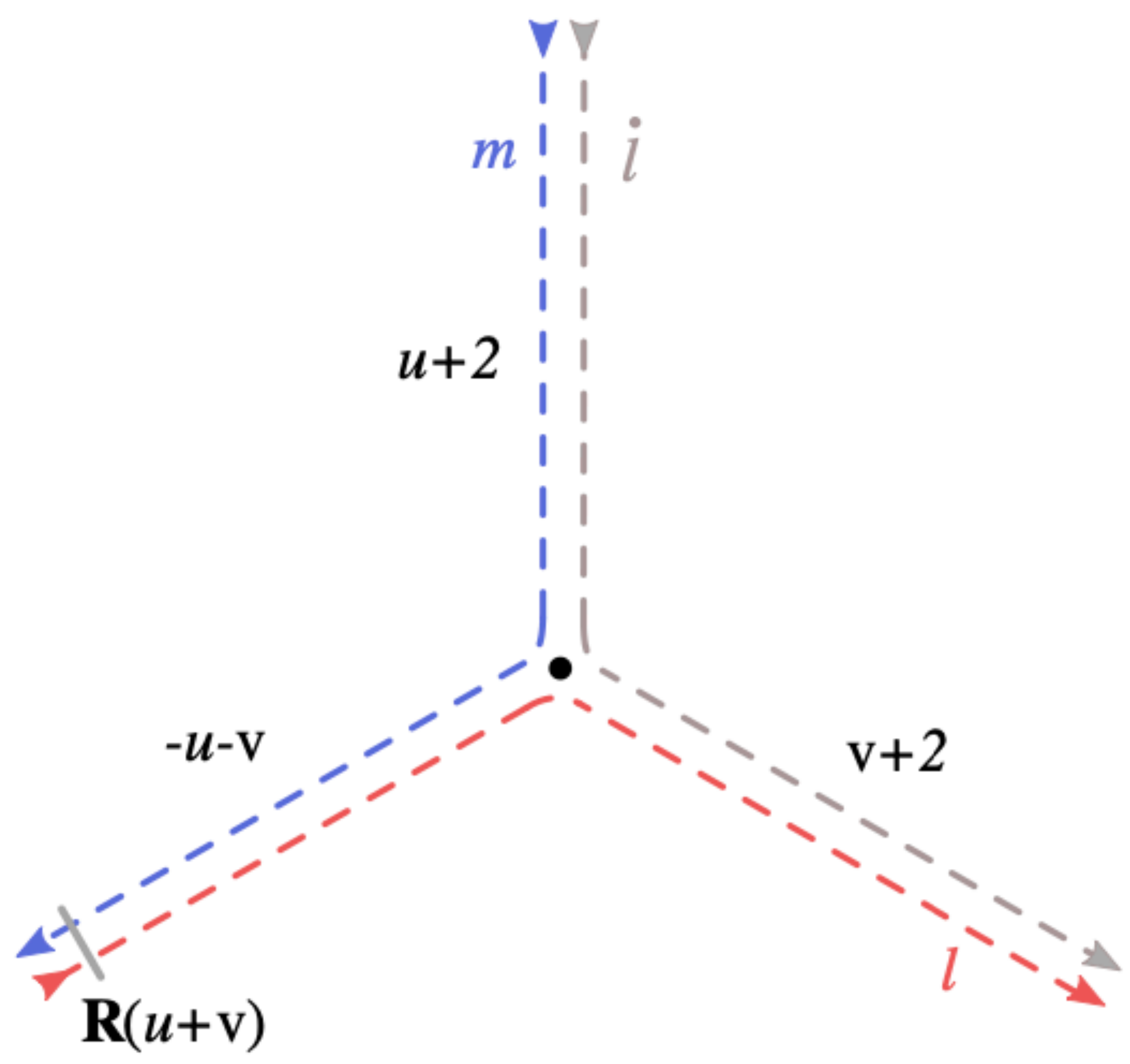
$$\Delta_{\pm} = \frac{\Delta_1 - \Delta_2}{2}$$

STAR–TRIANGLE IDENTITY

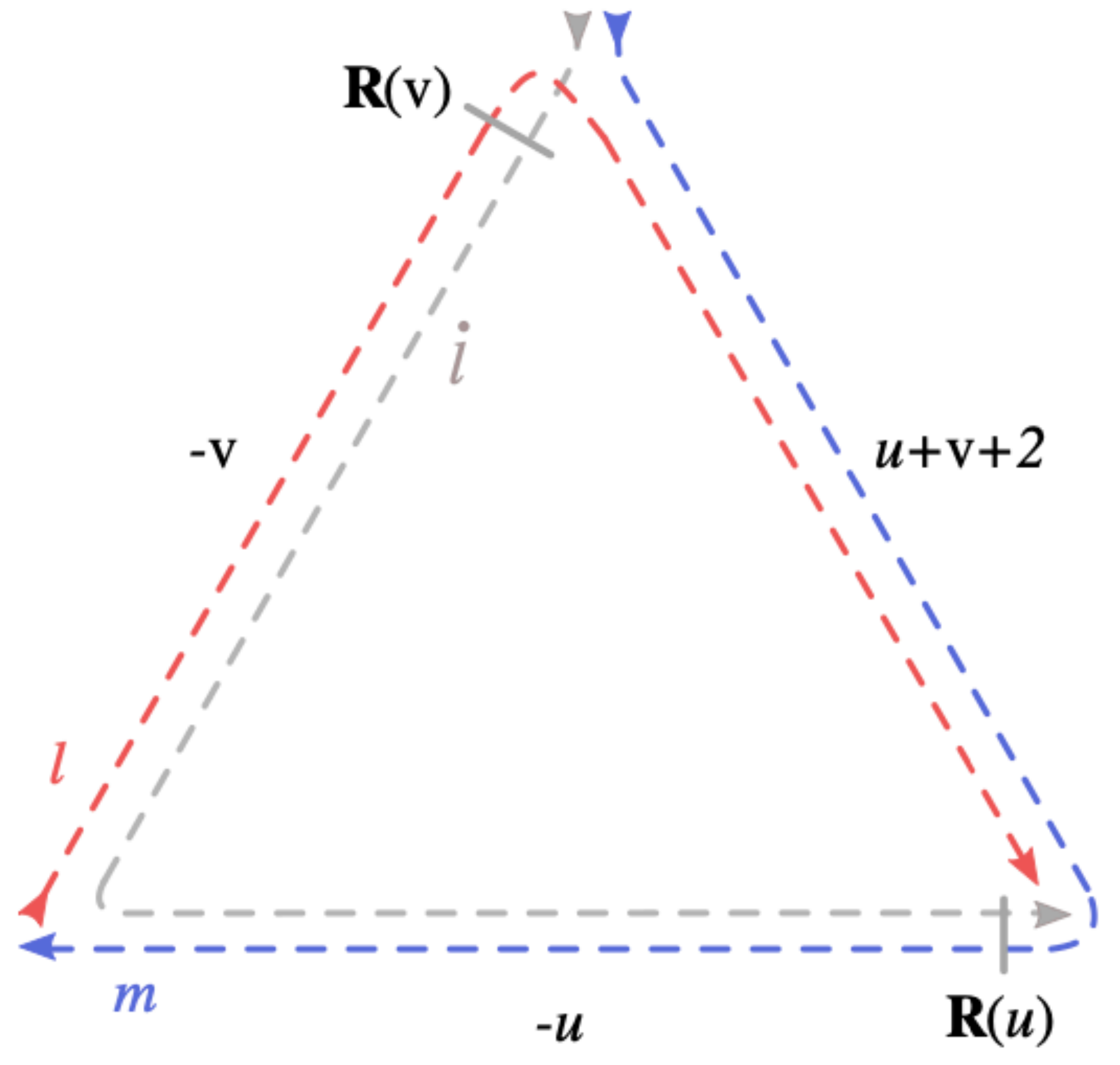
$$\frac{[\mathbf{x}]^{\ell}[\overline{\mathbf{x}}]^{\dot{\ell}}}{(x^2)^{\Delta}}$$

$$(\mathbf{R}_{mn})^{\mathbf{bd}}_{\mathbf{ac}}(u)$$



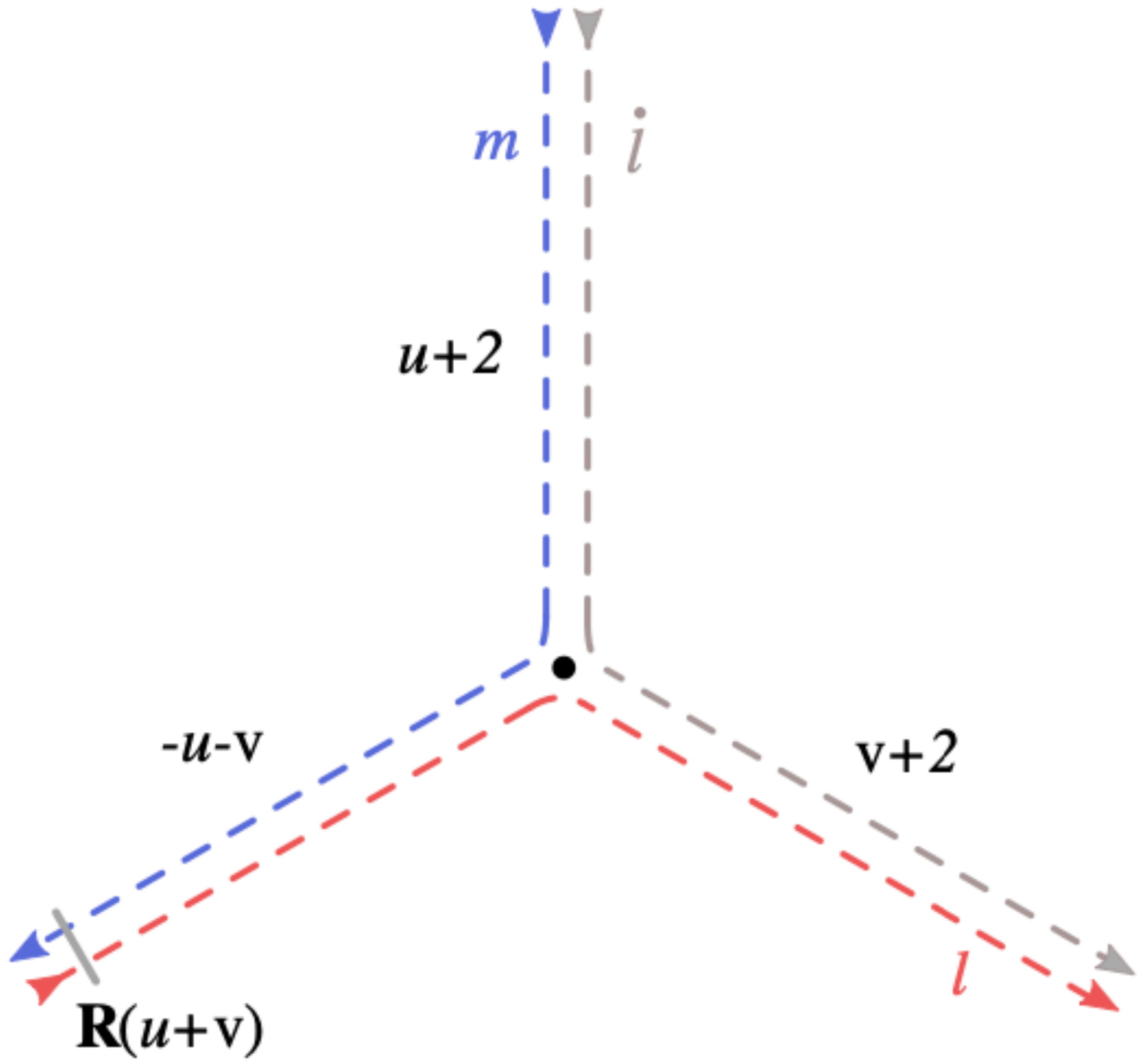


$$= \frac{a_{\dot{\ell}m}(u)\,a_{\ell\dot{\ell}}(v)}{a_{\ell m}(u+v)} \times$$

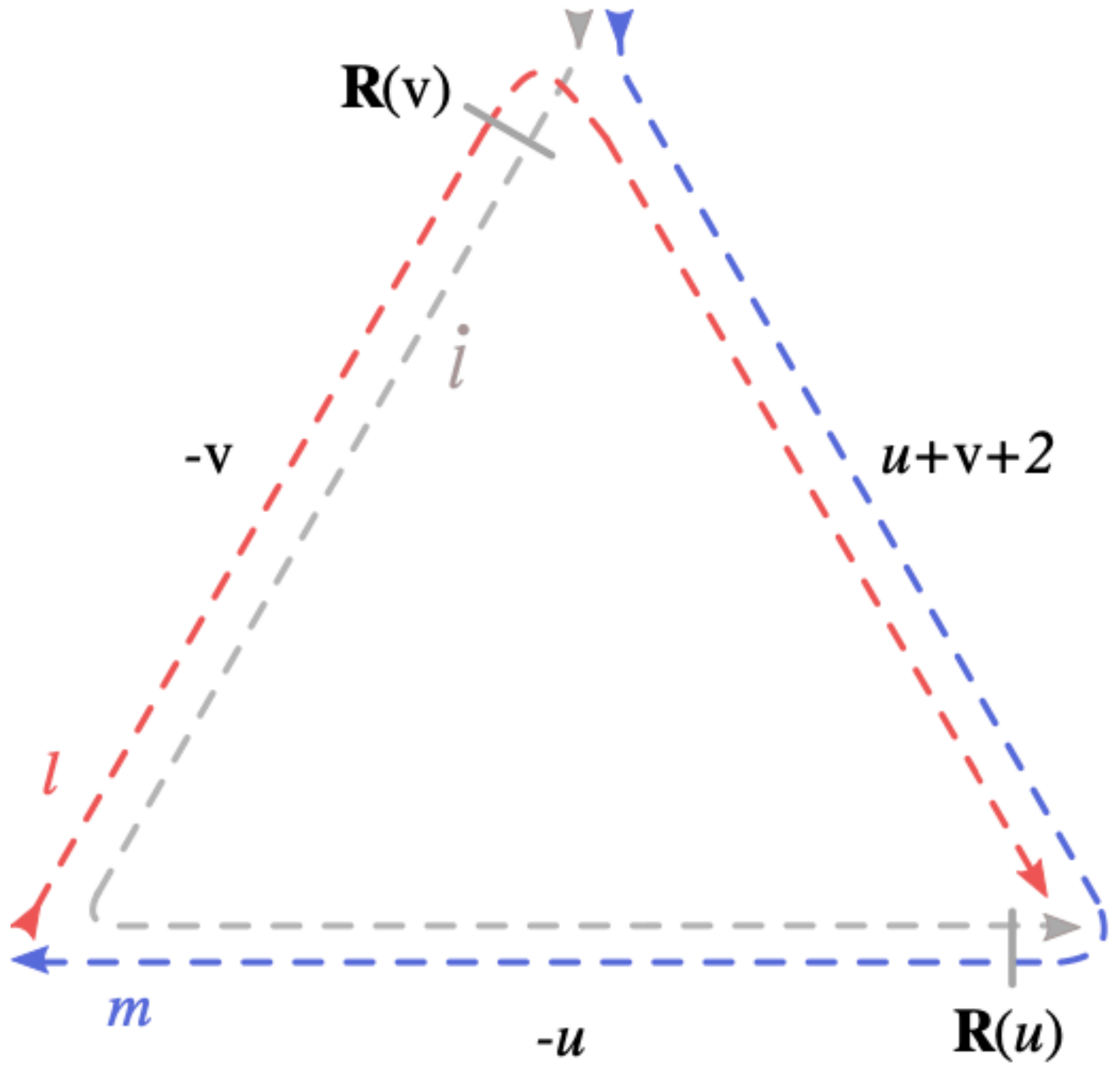


- Yang-Baxter equation is a ***consequence*** of STR! [V. Bazhanov's & V. Kazakov's talks]

STAR-TRIANGLE IDENTITY



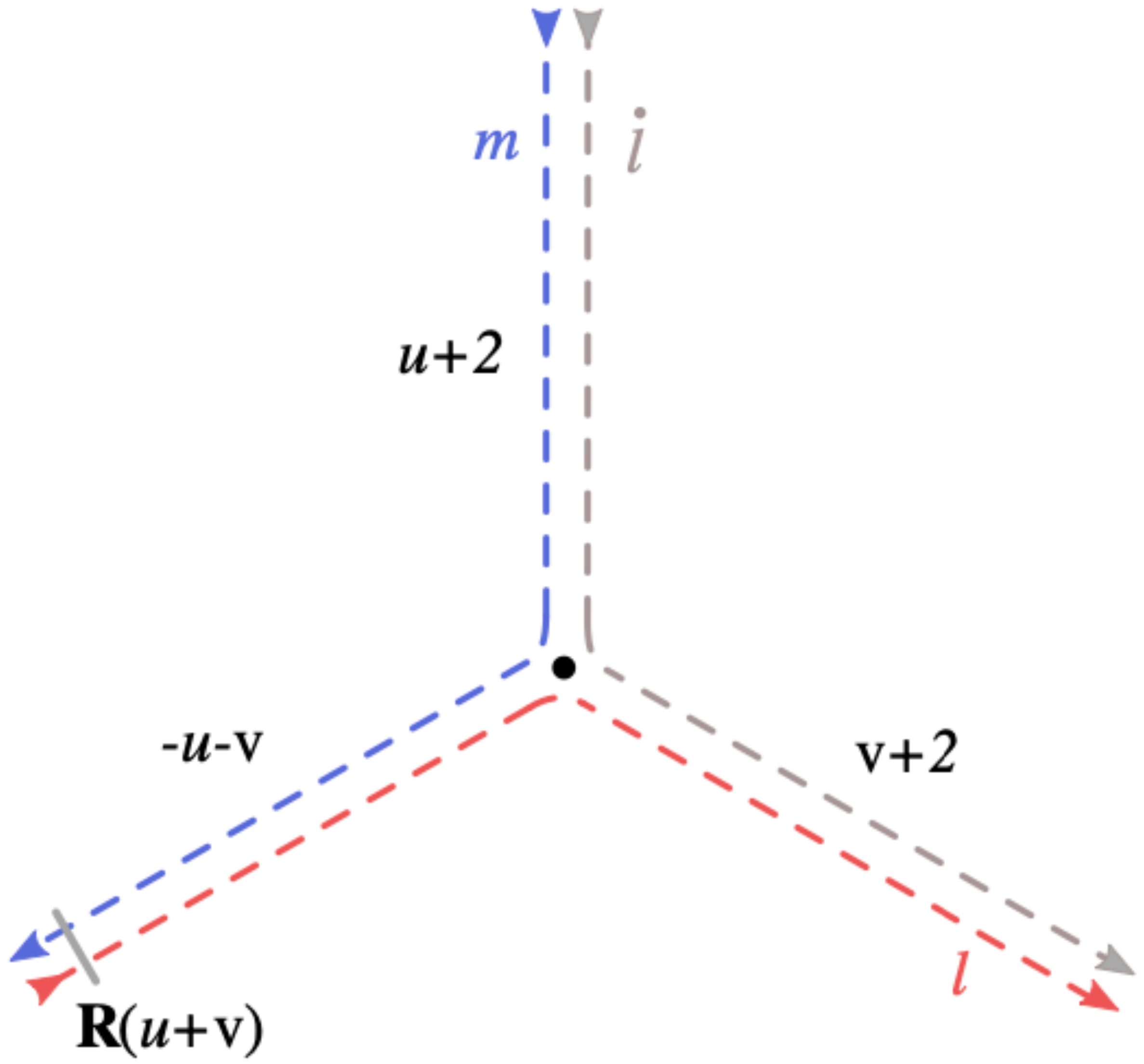
$$= \frac{a_{\dot{\ell} m}(u)\, a_{\ell \dot{\ell}}(v)}{a_{\ell m}(u+v)} \times$$



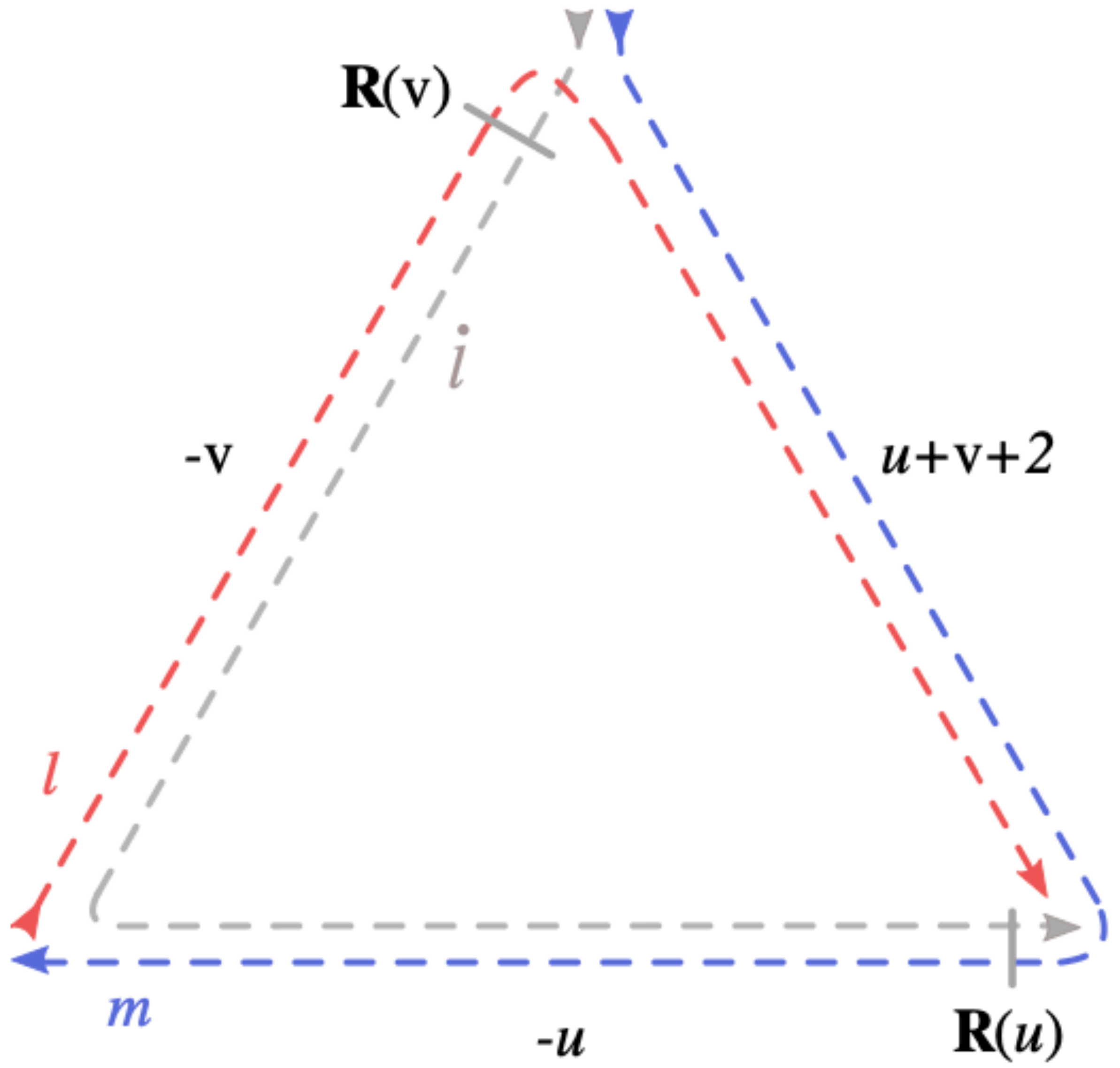
$$\hat{p}^{2u}\, [\overline{\mathbf{p}}]^{\dot{\ell}} \mathbf{R}_{m\dot{\ell}}(u) [\mathbf{p}]^{m}\; x^{2(u+v)}\, [\overline{\mathbf{x}}]^{m} \mathbf{R}_{m\ell}(u+v) [\mathbf{x}]^{\ell}\; \hat{p}^{2v}\, [\overline{\mathbf{p}}]^{\ell} \mathbf{R}_{\ell\dot{\ell}}(v) [\mathbf{p}]^{\dot{\ell}} \; =$$

$$x^{2v}\, [\overline{\mathbf{x}}]^{\dot{\ell}} \mathbf{R}_{\ell\dot{\ell}}(v) [\mathbf{x}]^{\ell}\; \hat{p}^{2(u+v)}\, [\overline{\mathbf{p}}]^{\ell} \mathbf{R}_{m\ell}(u+v) [\mathbf{p}]^{m}\; x^{2u}\, [\overline{\mathbf{x}}]^{m} \mathbf{R}_{m\dot{\ell}}(u) [\mathbf{x}]^{\dot{\ell}} \; .$$

STAR-TRIANGLE IDENTITY



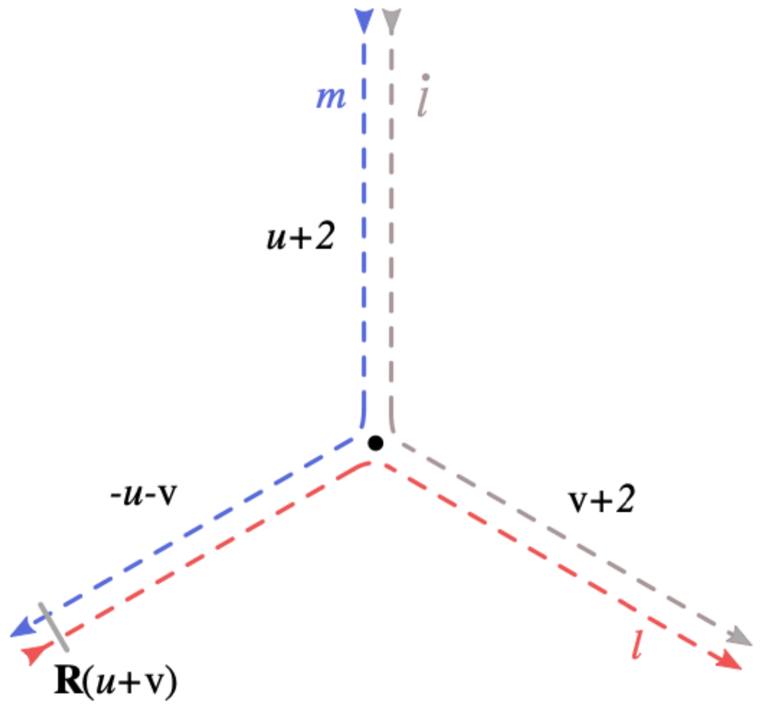
$$= \frac{a_{\dot\ell m}(u)\,a_{\ell\dot\ell}(v)}{a_{\ell m}(u+v)} \times$$



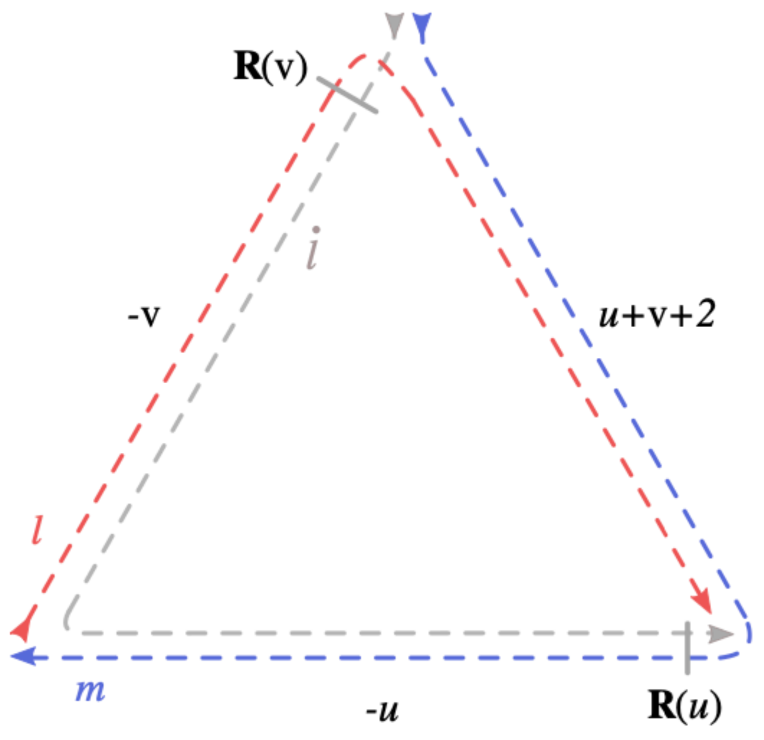
$$\int d^4y\, \frac{[\overline{\mathbf{x}-\mathbf{y}}]^{\dot\ell}[\mathbf{x}-\mathbf{y}]^m}{(x-y)^{2(u+2)}}\, \frac{[\overline{\mathbf{y}}]^m \mathbf{R}_{m\ell}(u+v)[\mathbf{y}]^\ell}{y^{-2(u+v)}}\, \frac{[\overline{\mathbf{y}-\mathbf{z}}]^\ell[\mathbf{y}-\mathbf{z}]^{\dot\ell}}{(y-z)^{2(v+2)}} =$$

$$= \pi^2 \frac{a_{\dot\ell m}(u)\,a_{\ell\dot\ell}(v)}{a_{\ell m}(u+v)}\, \frac{[\overline{\mathbf{x}}]^{\dot\ell}\mathbf{R}_{\ell\dot\ell}(v)[\mathbf{x}]^\ell}{x^{-2v}}\, \frac{[\overline{\mathbf{x}-\mathbf{z}}]^\ell[\mathbf{x}-\mathbf{z}]^m}{(x-z)^{2(u+v+2)}}\, \frac{[\overline{\mathbf{z}}]^m\mathbf{R}_{m\dot\ell}(u)[\mathbf{z}]^{\dot\ell}}{z^{-2u}}$$

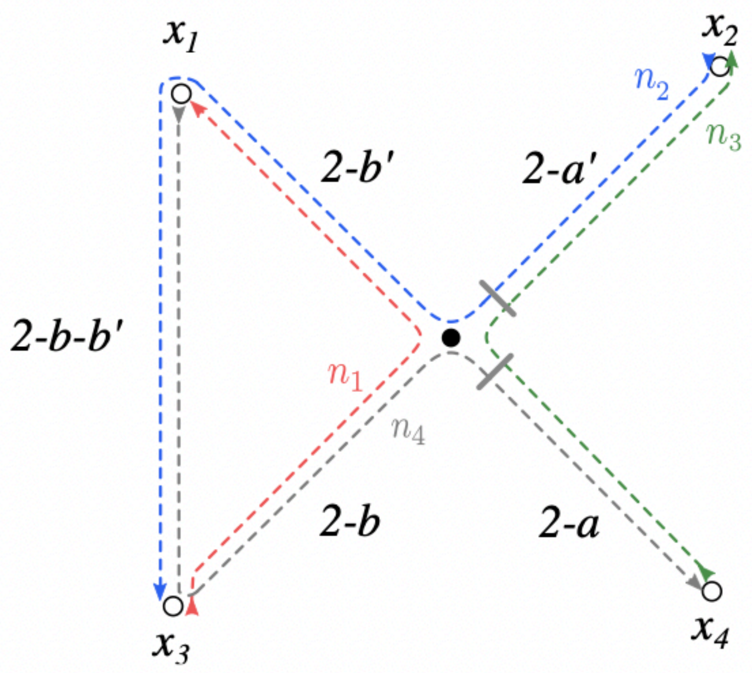
STAR–TRIANGLE IDENTITY



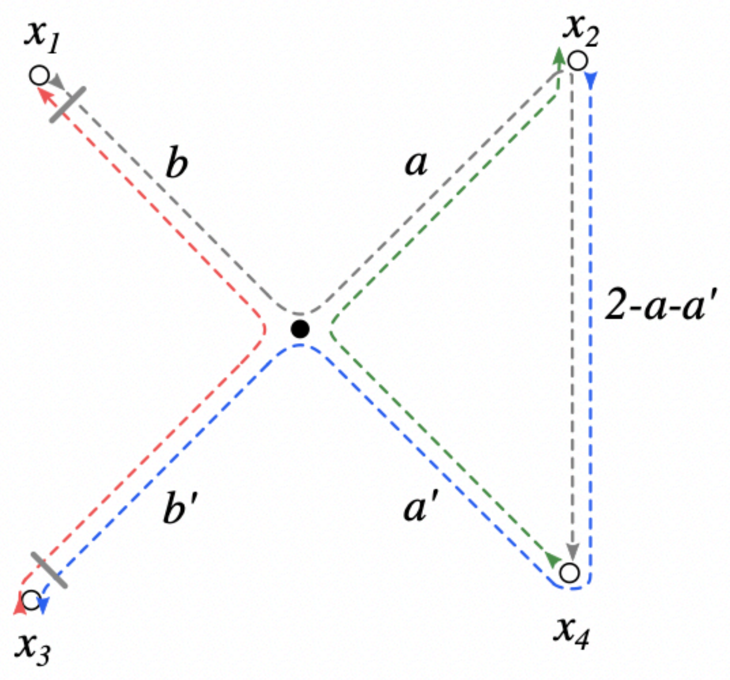
$$= \frac{a_{\dot{\ell} m}(u)\, a_{\ell \dot{\ell}}(v)}{a_{\ell m}(u+v)} \times$$



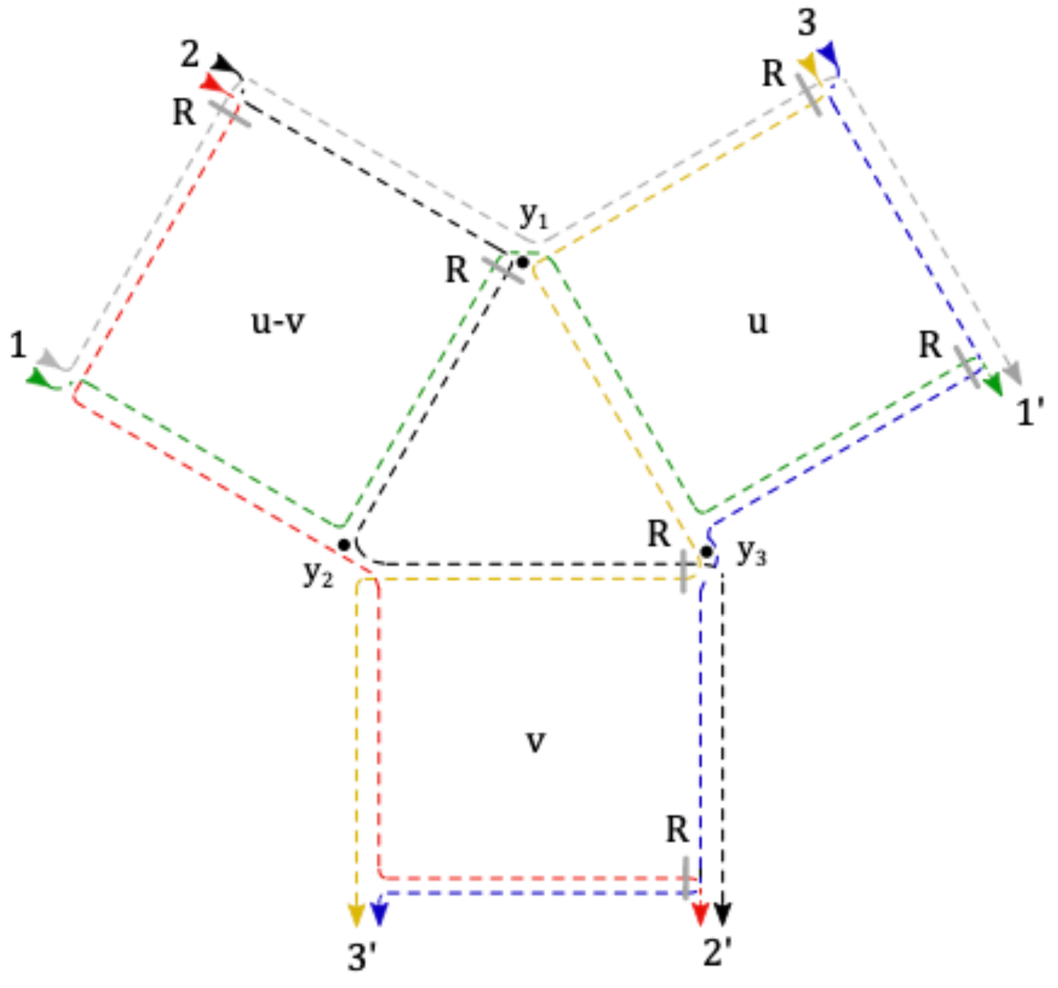
STAR–STAR RELATION

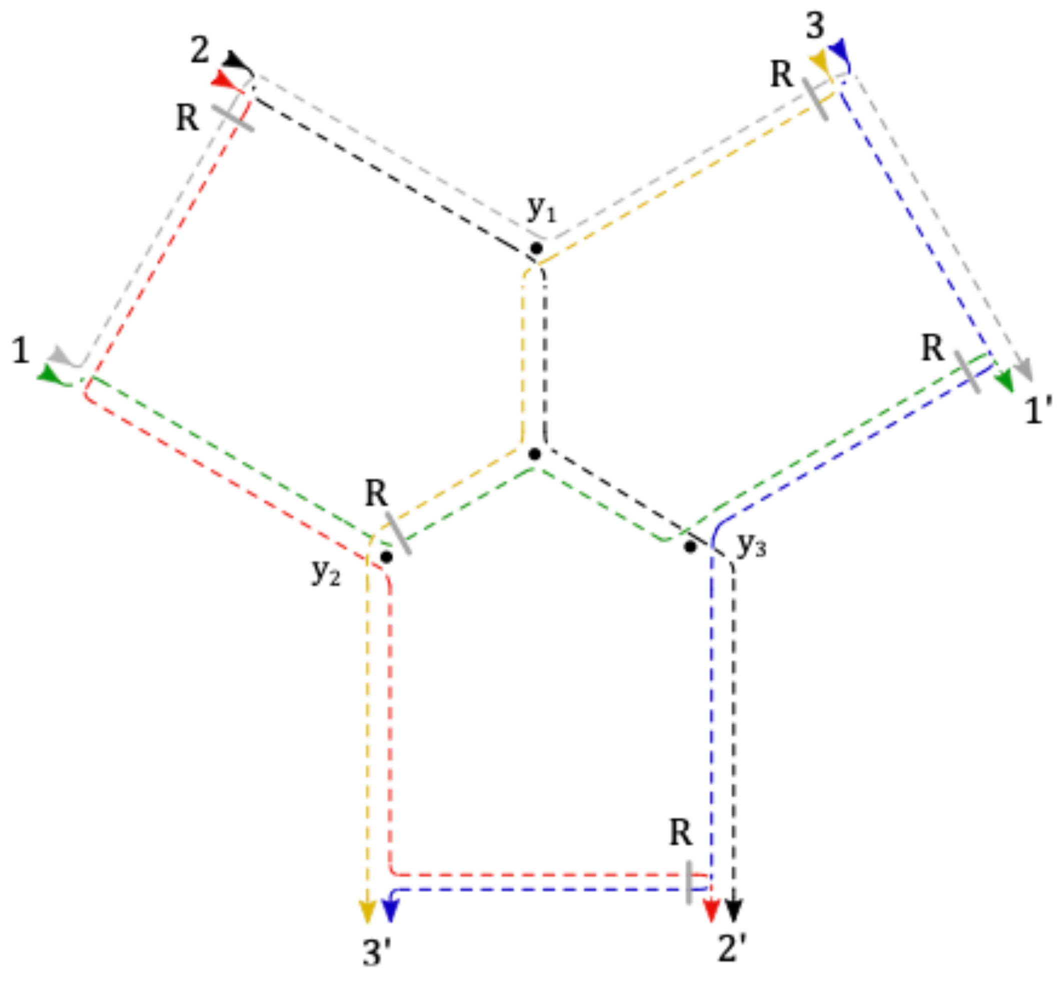


$$= \frac{a_{n_3 n_2}(a'-2)\; a_{n_3 n_4}(a-2)}{a_{n_1 n_2}(-b')\; a_{n_1 n_4}(-b)} \times$$



YANG–BAXTER EQUATION

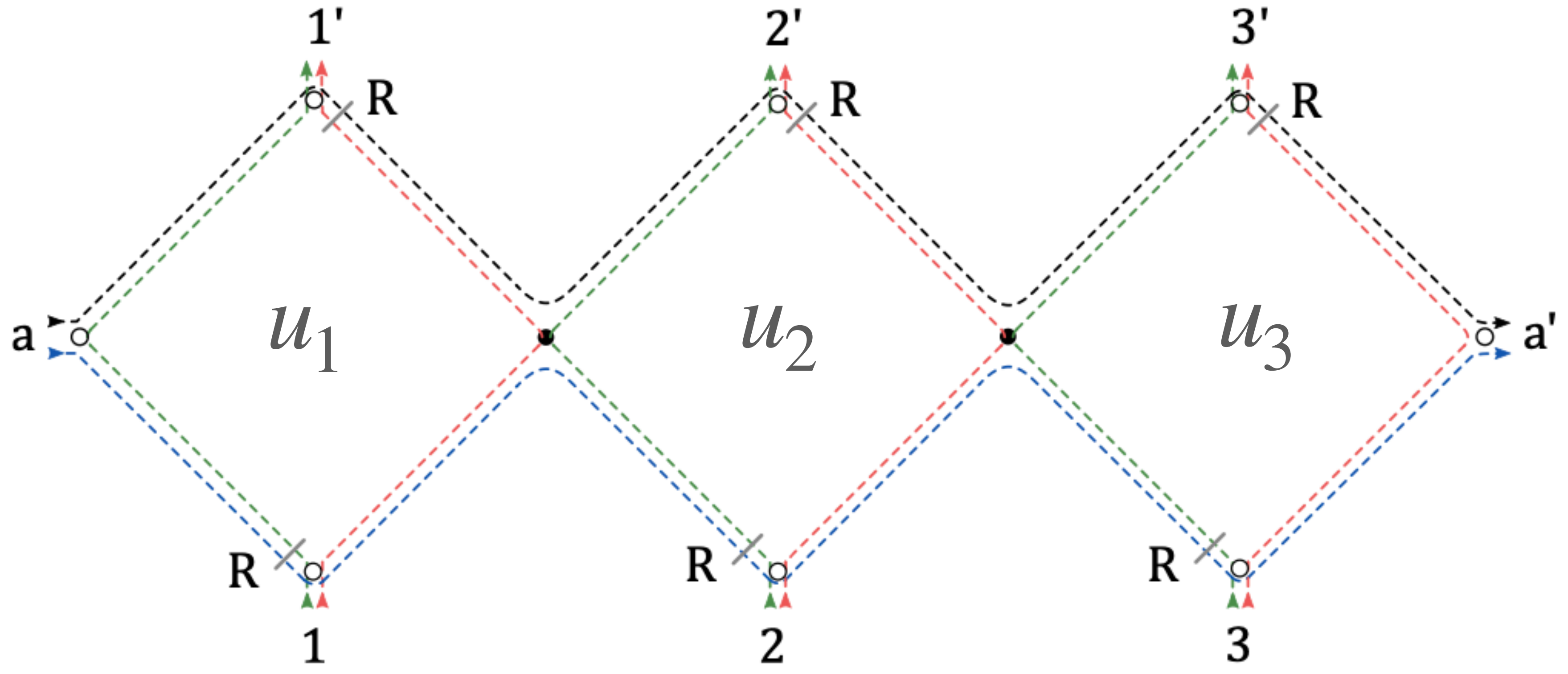


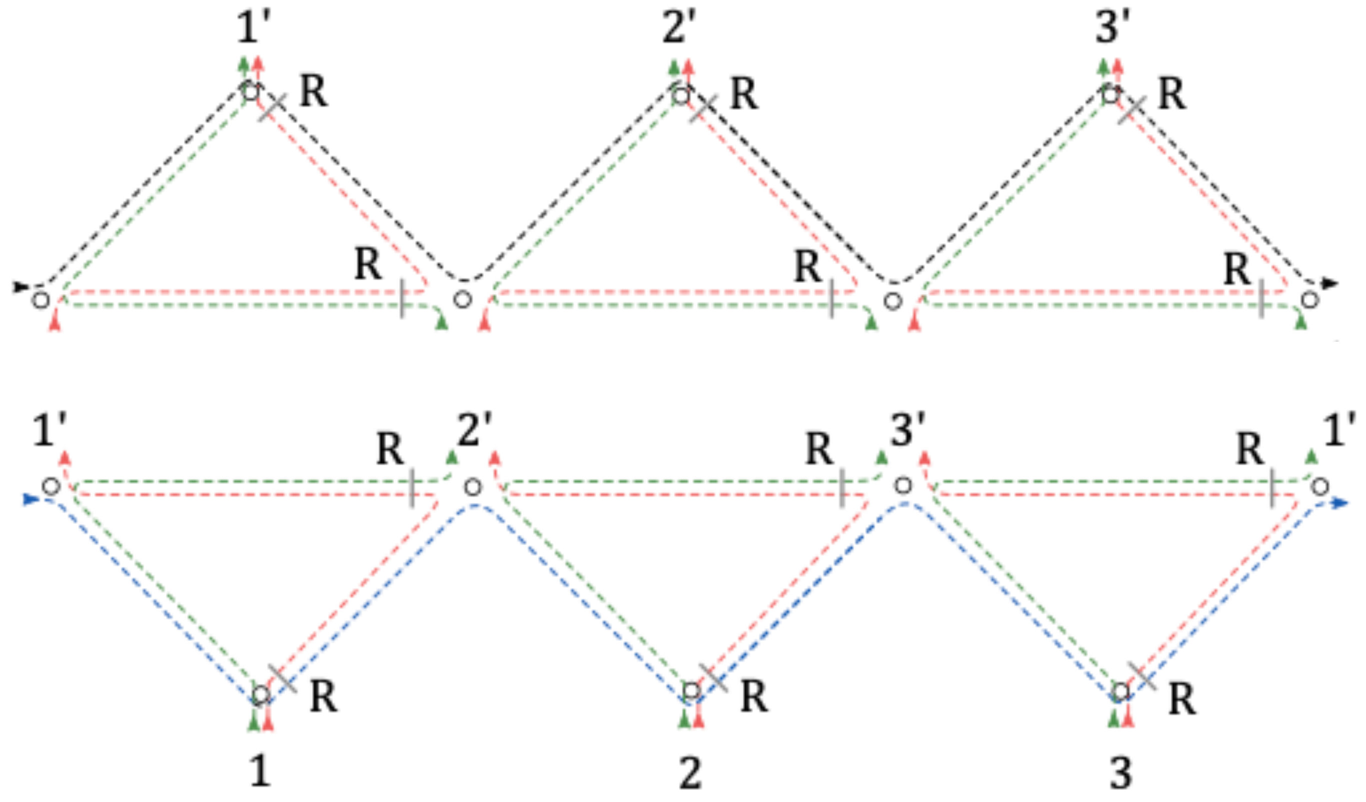


FACTORIZATION, RTT, TRT*

- Transfer matrix

$$\mathbb{T}_{1,2,3,a}(u_1, u_2, u_3) =$$





$$\mathbb{T}_{1,\ldots,L,a}(u) = \mathcal{R}_{1a}(u+\theta_1)\mathcal{R}_{2a}(u+\theta_2)\cdots\mathcal{R}_{La}(u+\theta_L)$$

$$\mathcal{R}_{ab}(u-v)\mathbb{T}_{1,\ldots,L,a}(u)\mathbb{T}_{1,\ldots,L,b}(v) = \mathbb{T}_{1,\ldots,L,a}(u)\,\mathbb{T}_{1,\ldots,L,b}(v)\,\mathcal{R}_{ab}(u-v)$$

RTT

$$\mathcal{R}_{12}(u)\mathcal{R}_{32}(u+v^*)\mathcal{R}_{13}(v)^\dagger = \mathcal{R}_{13}(v)^\dagger\mathcal{R}_{32}(u+v^*)\mathcal{R}_{12}(u)$$

$$\mathbb{T}_{1,\ldots,L,a}(u)\mathcal{R}_{ba}(u+v^*+\theta+\theta^*)\,\mathbb{T}_{1,\ldots,L,b}(v)^\dagger = \mathbb{T}_{1,\ldots,L,b}(v)^\dagger\,\mathcal{R}_{ba}(u+v^*+\theta+\theta^*)\,\mathbb{T}_{1,\ldots,L,a}(u)$$

TRT*

INTEGRABILITY

$$\mathcal{R}_{ab}(u-v)\mathbb{T}_{1,\ldots,L,a}(u)\mathbb{T}_{1,\ldots,L,b}(v) = \mathbb{T}_{1,\ldots,L,a}(u)\,\mathbb{T}_{1,\ldots,L,b}(v)\,\mathcal{R}_{ab}(u-v)$$

RTT

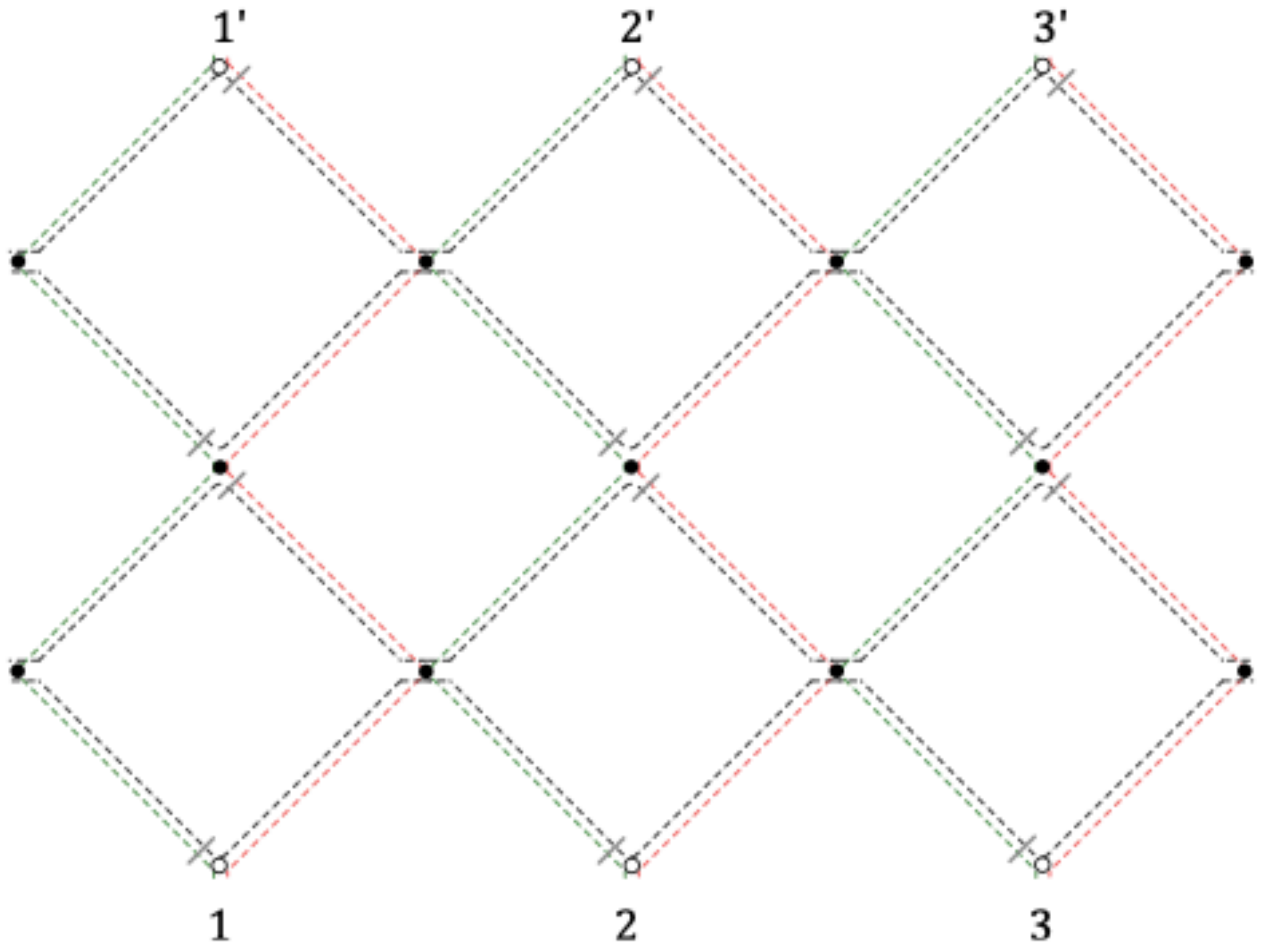
$$\mathbb{T}_{1,\ldots,L,a}(u)\mathcal{R}_{ba}(u+v^*+\theta+\theta^*)\,\mathbb{T}_{1,\ldots,L,b}(v)^\dagger = \mathbb{T}_{1,\ldots,L,b}(v)^\dagger\,\mathcal{R}_{ba}(u+v^*+\theta+\theta^*)\,\mathbb{T}_{1,\ldots,L,a}(u)$$

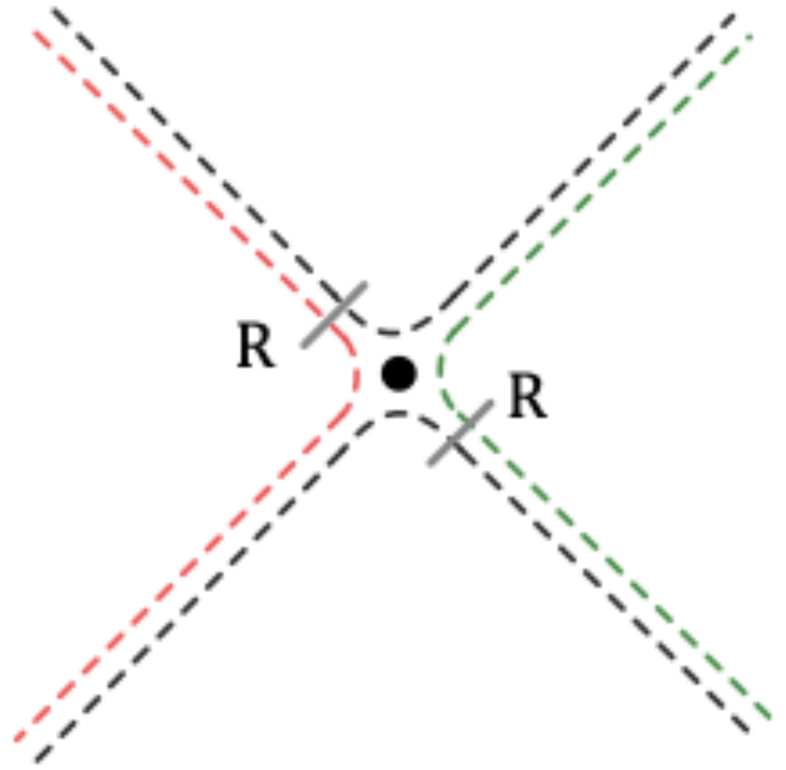
TRT*

$$\left[t^{(a)}(u), t^{(b)}(v)\right] = 0$$

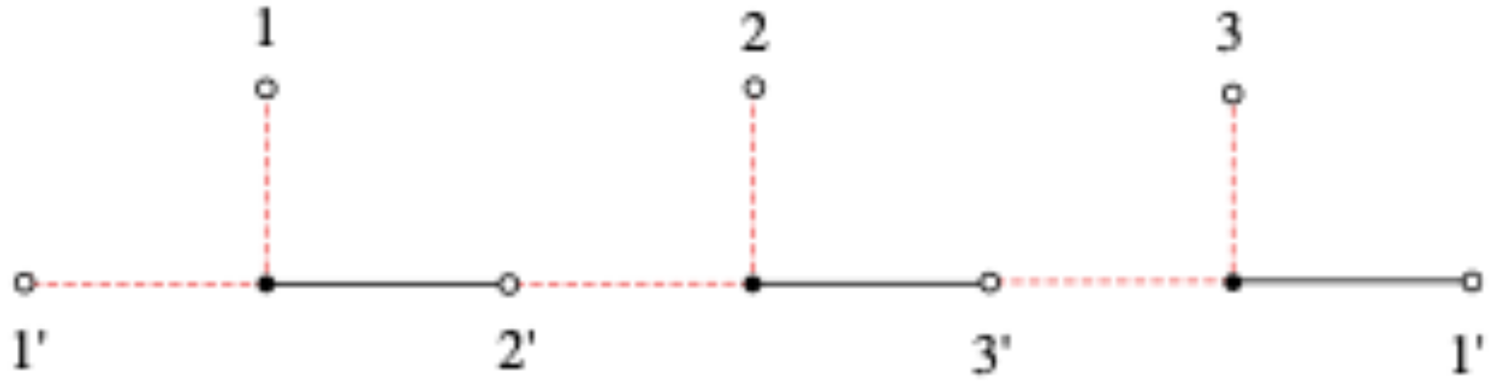
$$\left[t^{(a)}(u), (t^{(b)}(v))^\dagger\right] = 0$$

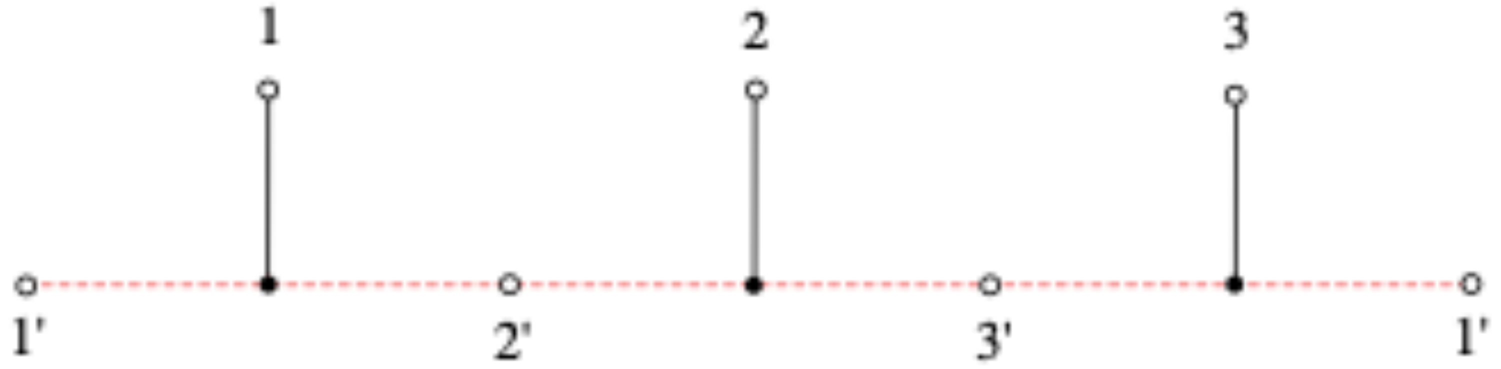
GENERALISED INTEGRABLE FISHNETS

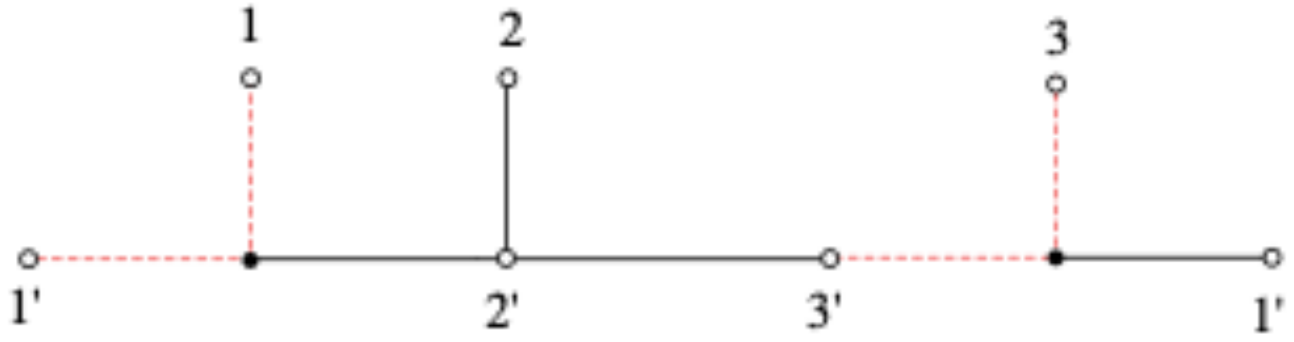




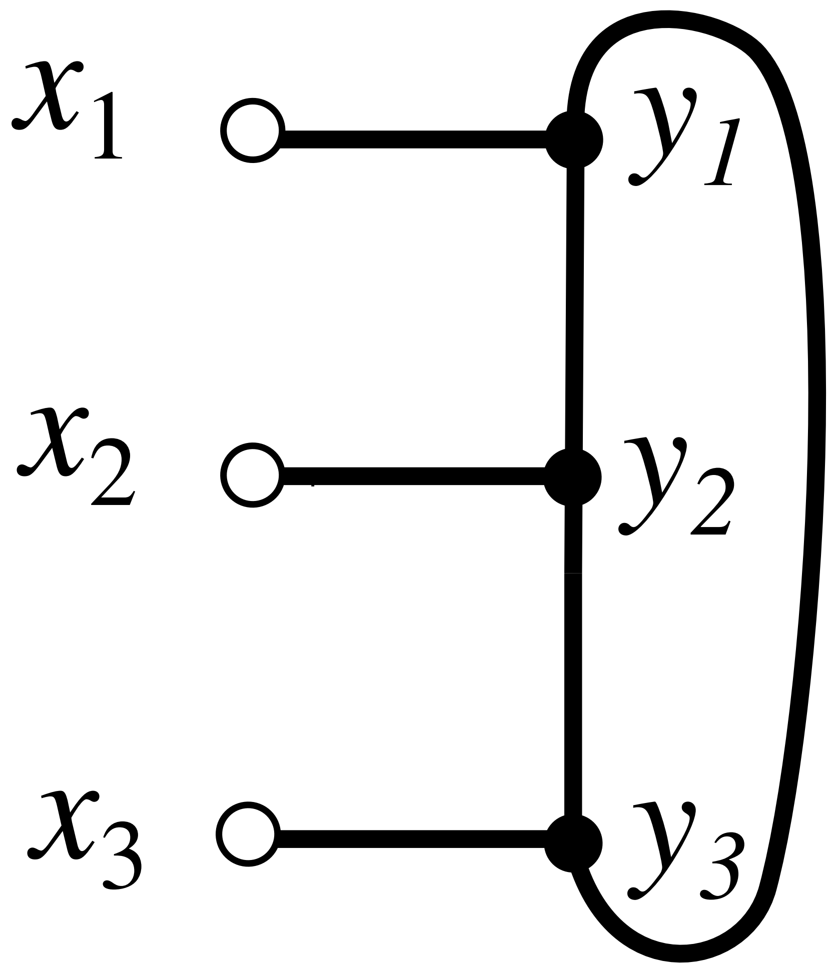
- ➤ Reductions

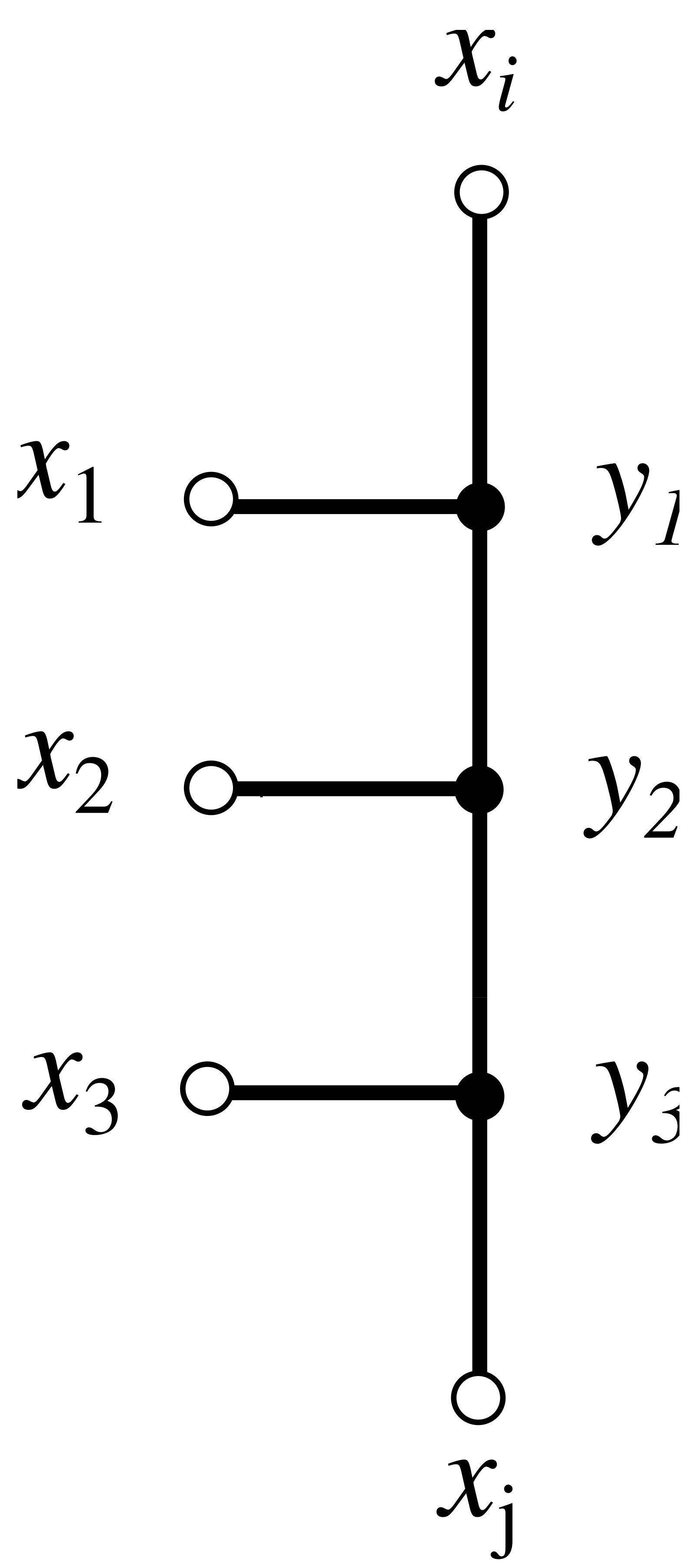






RECALL: DISK FISHNETS



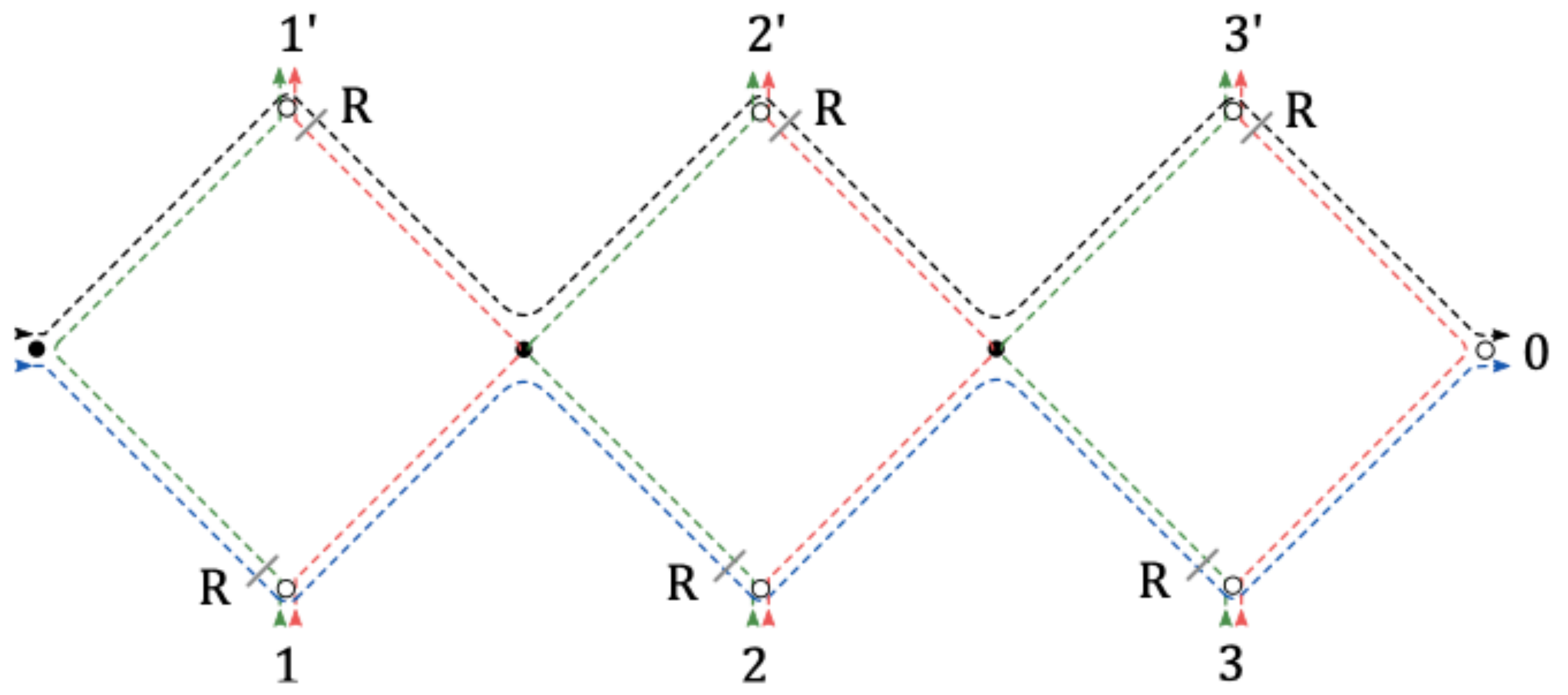


- Fixed boundaries !!!

FIXED BOUNDARIES: A SOLVABLE PROBLEM

$$\mathbf{Q}_{a,L}(u) = \sum_{\mathbf{a},\dot{\mathbf{a}}} \int d^4x_a \left[\mathbb{T}_{1,\dots,L,a}(u)\,\delta^{(4)}(x_a - x_0)\right]^{\mathbf{a}\dot{\mathbf{a}}}_{\mathbf{a}\dot{\mathbf{a}}} =$$

$$= \sum_{\mathbf{a}\,\dot{\mathbf{a}}} \int d^4x_a \left[\mathcal{R}_{1a}(u+\theta_1)\mathcal{R}_{2a}(u+\theta_2)\cdots\mathcal{R}_{La}(u+\theta_L)\,\delta^{(4)}(x_a - x_0)\right]^{\mathbf{a}\dot{\mathbf{a}}}_{\mathbf{a}\dot{\mathbf{a}}}$$



$$\Gamma_{x_0}(x|y)^{\mathbf{bd}}_{\mathbf{ac}} = \delta^{\mathbf{b}}_{\mathbf{a}}\delta^{\mathbf{d}}_{\mathbf{c}}\delta^{(4)}(x - x_0) \qquad \Gamma^{\dagger}_{x_0}(x|y)^{\mathbf{bd}}_{\mathbf{ac}} = \Gamma_{x_0}(y|x)^{\mathbf{ac}}_{\mathbf{bd}}$$

$$[\mathcal{R}_{ab}(u), \Gamma_a\Gamma_b] = 0\,, \quad \Gamma^{\dagger}_a\mathcal{R}_{ab}(u)\Gamma_b = \Gamma_b\mathcal{R}_{ab}(u)\Gamma^{\dagger}_a.$$

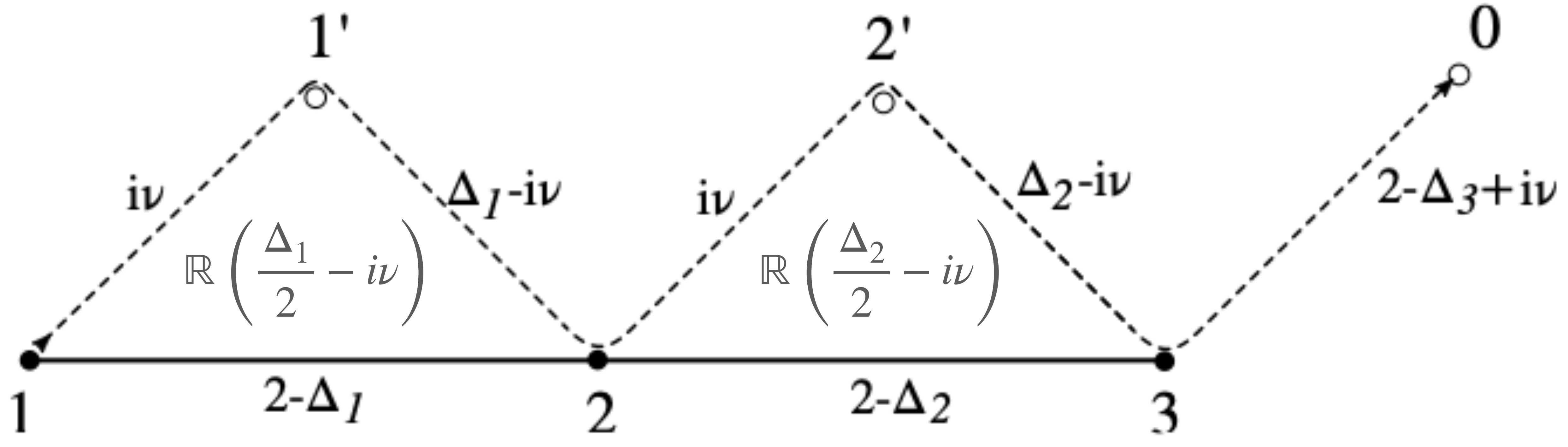
$$[\mathbf{Q}_a(u), \mathbf{Q}_b(v)] = 0\,, \quad [\mathbf{Q}_a(u), \mathbf{Q}_b(v)^{\dagger}] = 0$$

EIGENFUNCTIONS (BI-SCALAR FISHNET)

$$\mathbf{Q}_{a,k}(u)\mathbf{\Lambda}_k(Y|\eta,\bar{\eta}) = q_{a,k}(Y)\,\mathbf{\Lambda}_k(Y|\eta,\bar{\eta})\mathbf{Q'}_{a,k-1}(u)\,,$$

$$\mathbf{\Lambda}_1(Y) = \Psi_1(Y|x) = \frac{[(\mathbf{x}-\mathbf{x_0})]^n}{(x-x_0)^{2(2-\Delta_1+i\nu)}}$$

$$\mathbf{\Lambda}_k(Y) \equiv \mathbf{\Lambda}_k(n,\nu) = \mathbb{R}^{(n)}_{12}\left(\frac{\Delta_1}{2}-i\nu\right)\mathbb{R}^{(n)}_{23}\left(\frac{\Delta_2}{2}-i\nu\right)\cdots\mathbb{R}^{(n)}_{k-1k}\left(\frac{\Delta_{k-1}}{2}-i\nu\right)\frac{[(\mathbf{x_k}-\mathbf{x_0})]^n}{(x_k-x_0)^{2(2-\Delta_k+i\nu)}}$$

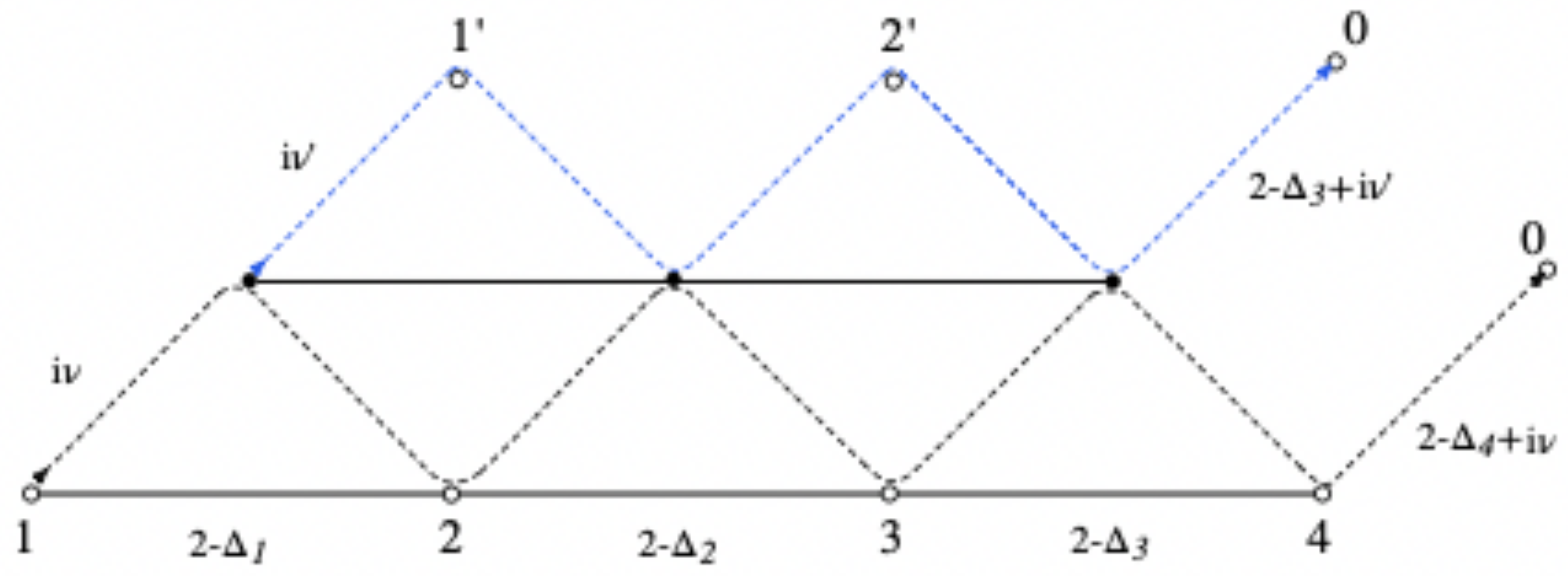


EIGENFUNCTIONS (BI-SCALAR FISHNET)

$$\mathbf{Q}_{a,k}(u)\mathbf{\Lambda}_k(Y|\eta,\bar{\eta}) = q_{a,k}(Y)\,\mathbf{\Lambda}_k(Y|\eta,\bar{\eta})\mathbf{Q'}_{a,k-1}(u)\,,$$

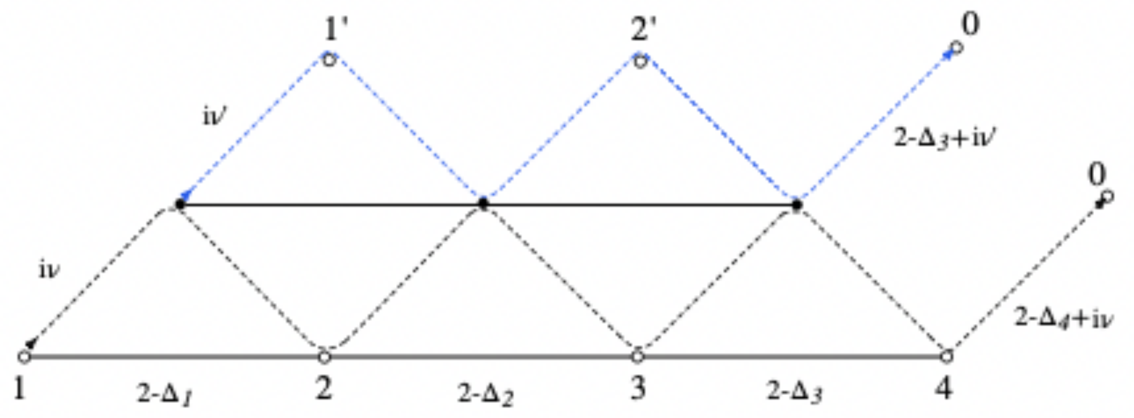
- Iterative construction: separation of variables

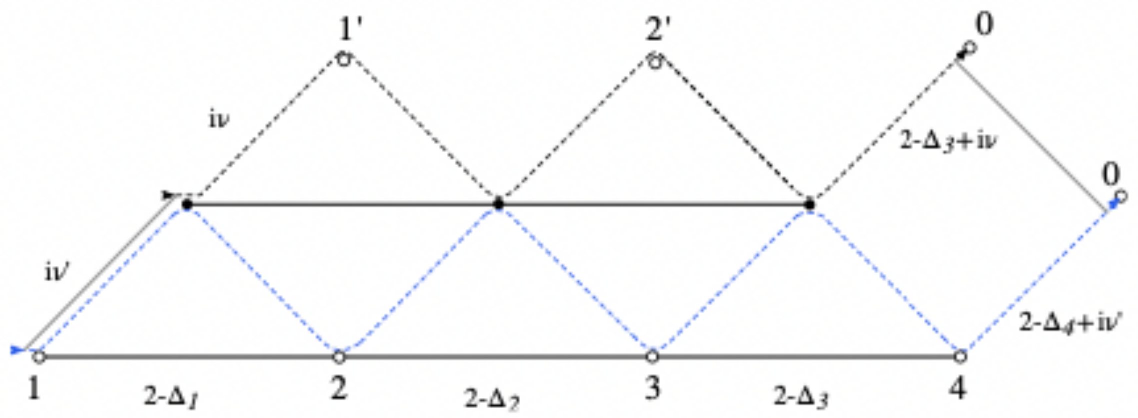
$$\Psi(\mathbf{Y}|\mathbf{x}) = \mathbf{\Lambda}_L(Y_L)\cdots\mathbf{\Lambda}_2(Y_2)\mathbf{\Lambda}_1(Y_1)\prod_{k=1}^{L} r_k(Y)^{k-1}\,.$$



- Exchange of layers: factorised scattering of excitations Y, Y'

$$\mathbf{\Lambda}_{k+1}(Y)\cdot\mathbf{\Lambda}_k(Y') = \frac{r_k(Y')}{r_k(Y)}\times\mathbf{R}(Y',Y)\mathbf{\Lambda}_{k+1}(Y')\cdot\mathbf{\Lambda}_k(Y)\mathbf{R}(Y,Y')$$

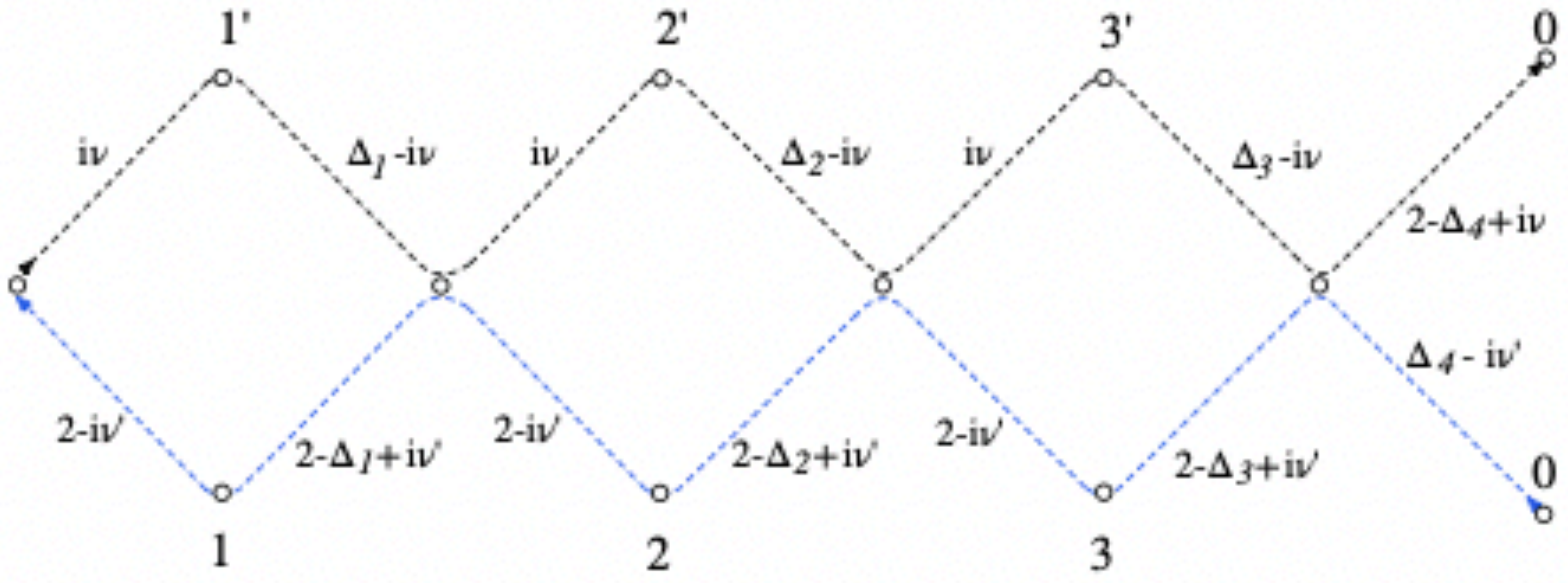


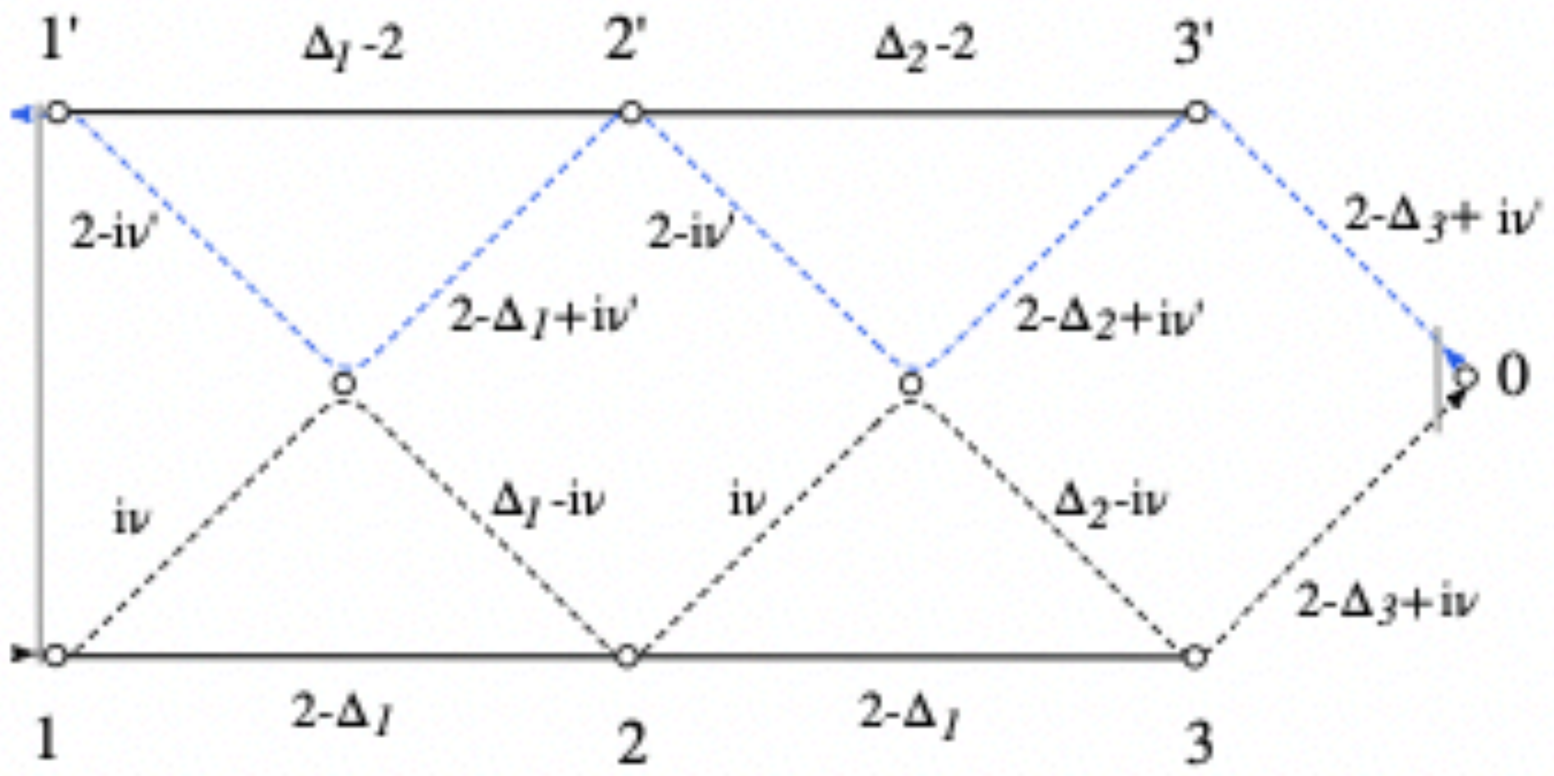


EIGENFUNCTIONS (BI-SCALAR FISHNET)

- Overlap of layers: factorised scattering of excitations Y, Y’

$$\bar{\mathbf{\Lambda}}_{k+1}(Y')\cdot\mathbf{\Lambda}_{k+1}(Y)=\frac{\pi^4}{\mu(Y,Y')}\frac{r_k(Y)}{r_k(Y')}\times\left[\mathbf{R}(Y',Y)^{t'}\mathbf{\Lambda}_k(Y)\cdot\bar{\mathbf{\Lambda}}_k(Y')^{t'}\mathbf{R}(Y,Y')^{t'}\right]^{t'}$$





- Non-factorisable measure (Vandermonde-like)

$$\mu(Y,Y')=\left|i(\nu-\nu')+\frac{n-n'}{2}\right|^2\left|1+i(\nu-\nu')+\frac{n+n'}{2}\right|^2$$

SPECTRAL (SOV) TRANSFORMATION

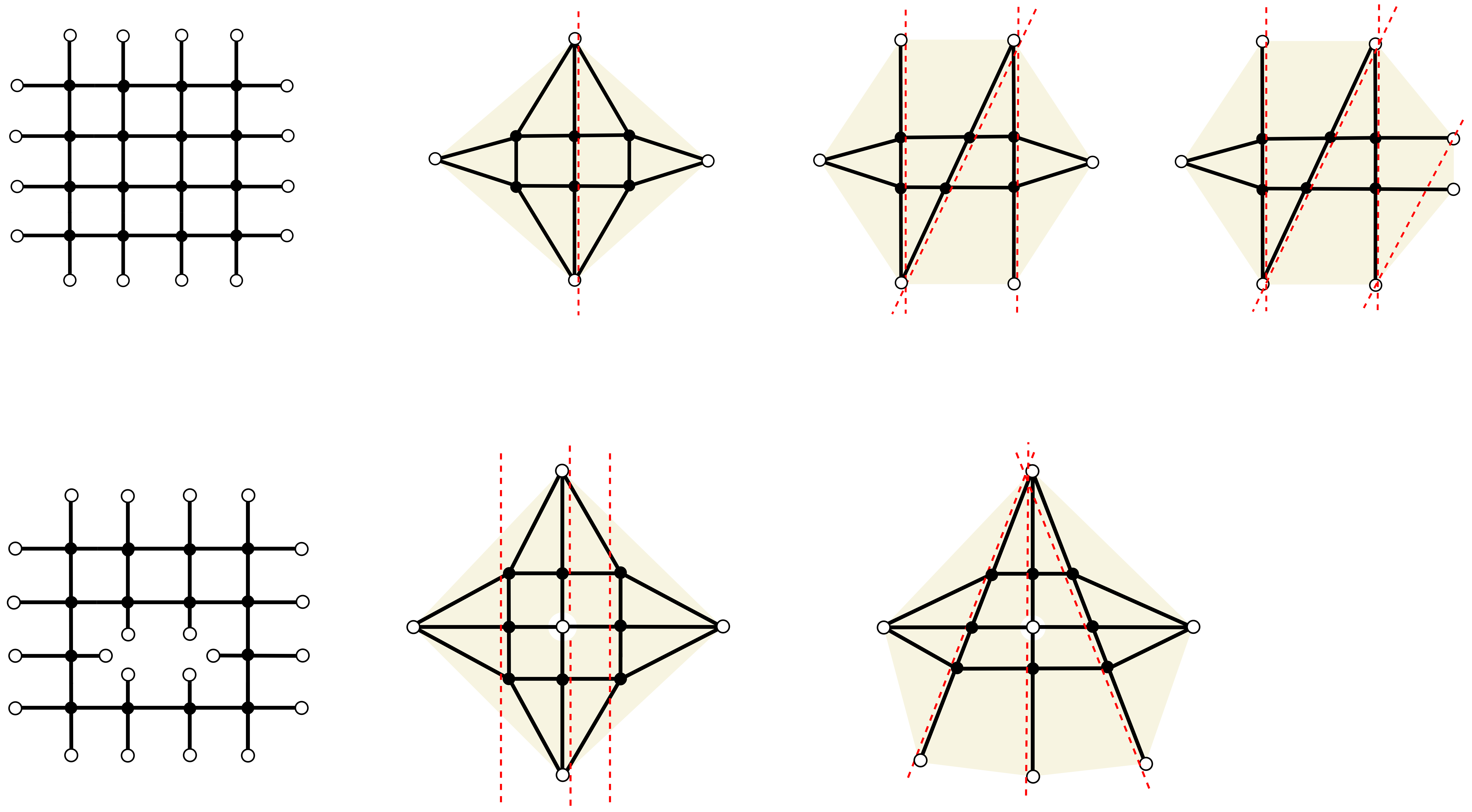
$$\mu(Y,Y') = \left|i(\nu-\nu') + \frac{n-n'}{2}\right|^2 \left|1 + i(\nu-\nu') + \frac{n+n'}{2}\right|^2$$

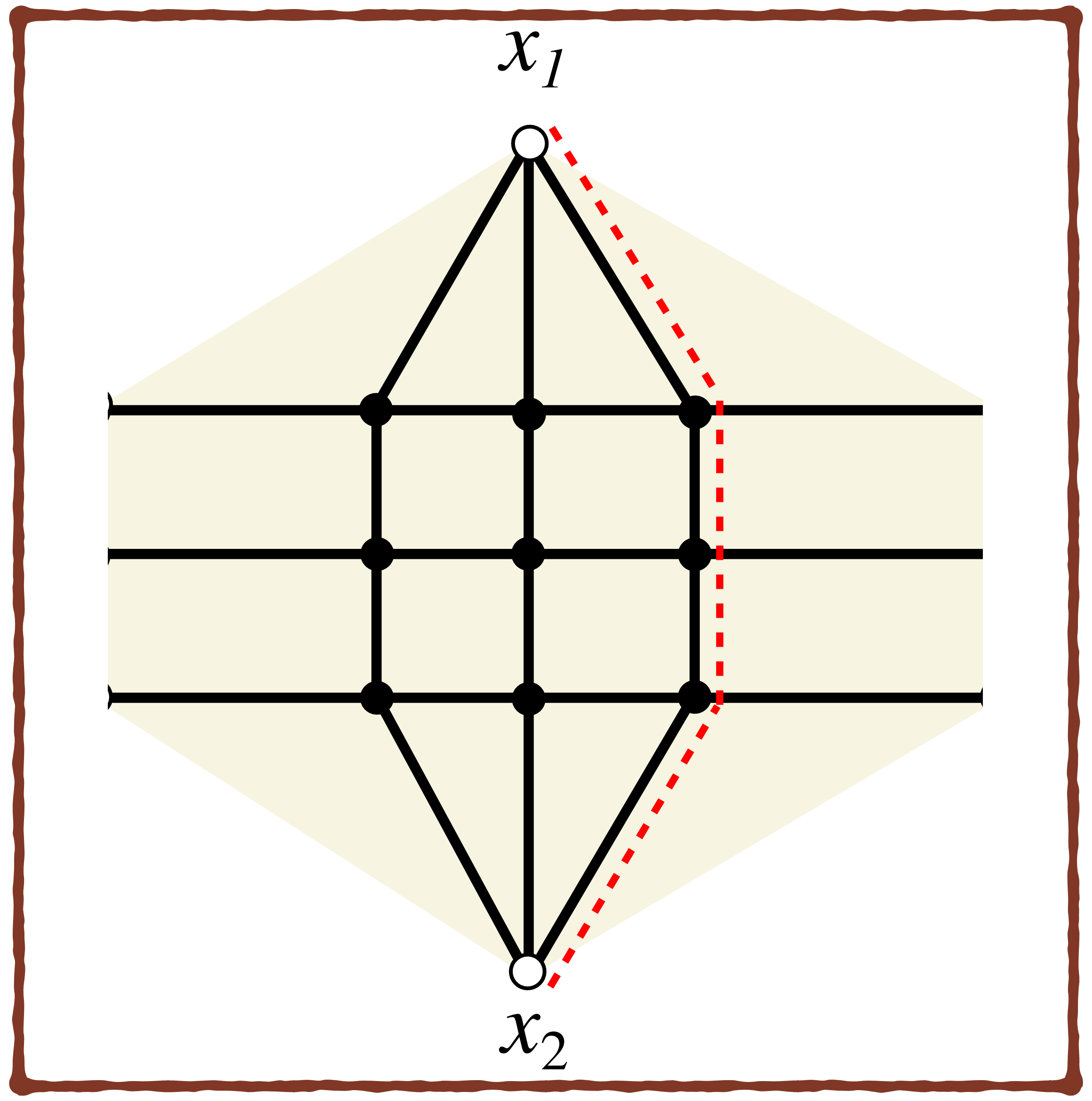
$$\rho(Y_1,\ldots,Y_L) = \prod_{j=1}^{L} \frac{(n_j+1)}{2\pi^{(2L+1)}} \prod_{k\neq j}^{L} \mu(Y_j,Y_k)$$

$$\mathcal{U} : \Phi(x_1,\ldots,x_L) \mapsto (\mathcal{U}\Phi)(\mathbf{Y}) = \int d^4x_1 \cdots d^4x_L \Psi(\mathbf{x}|\mathbf{Y})^* \Phi(\mathbf{x}) = \langle \Psi(\mathbf{Y}), \Phi \rangle_{\mathcal{V}} \,,$$

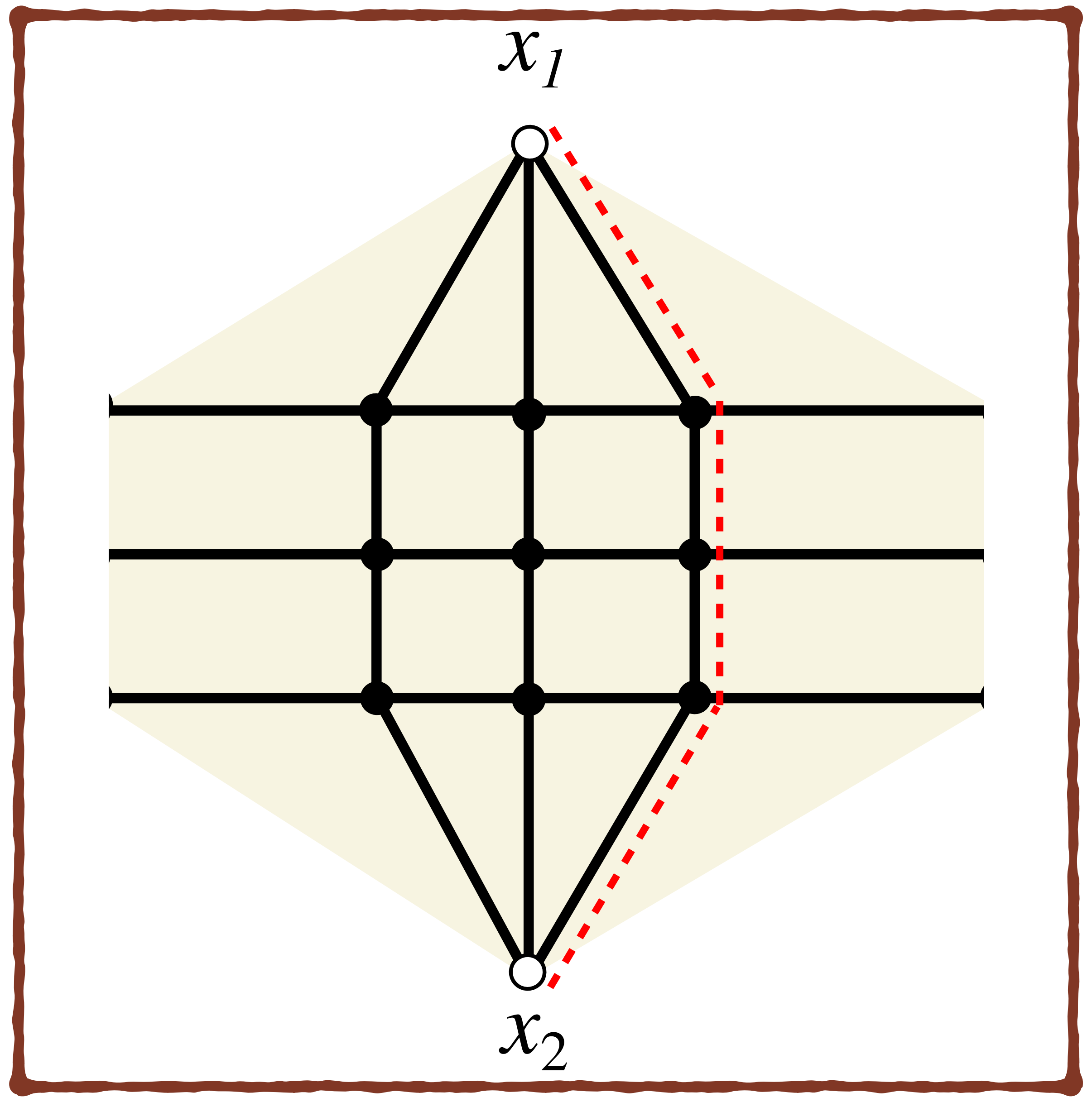
$$\mathcal{U}^{-1} : \widetilde{\Phi}(Y_1,\ldots,Y_L) \mapsto \left(\mathcal{U}^{-1}\widetilde{\Phi}\right)(\mathbf{x}) = \sum_{n_1,\ldots,n_L=0}^{\infty} \int d\nu_1 \cdots d\nu_L \, \rho(\mathbf{Y}) \widetilde{\Phi}(\mathbf{Y}) \Psi(\mathbf{x}|\mathbf{Y}) = \langle \widetilde{\Phi}, \Psi(\mathbf{x}) \rangle_{\widetilde{\mathcal{V}}} \,.$$

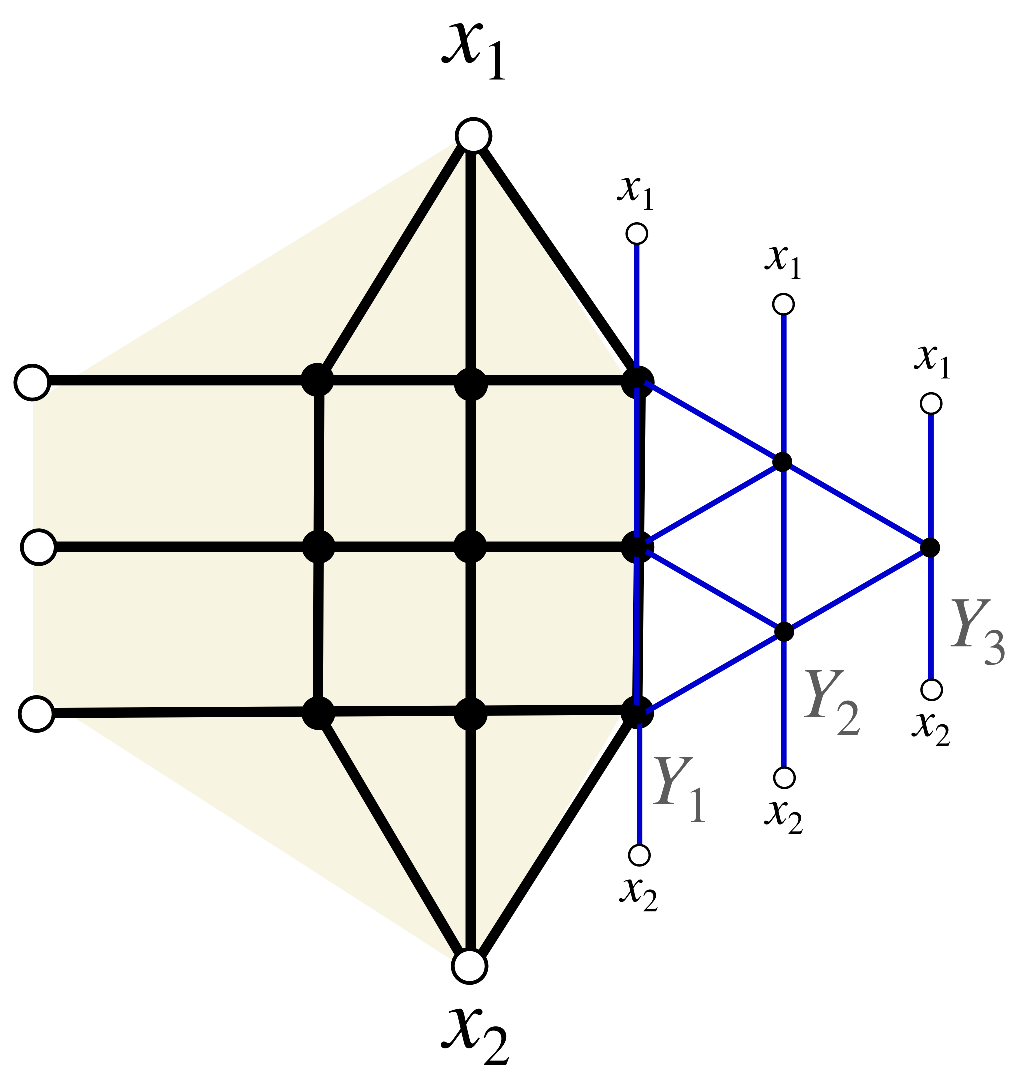
CUTTING

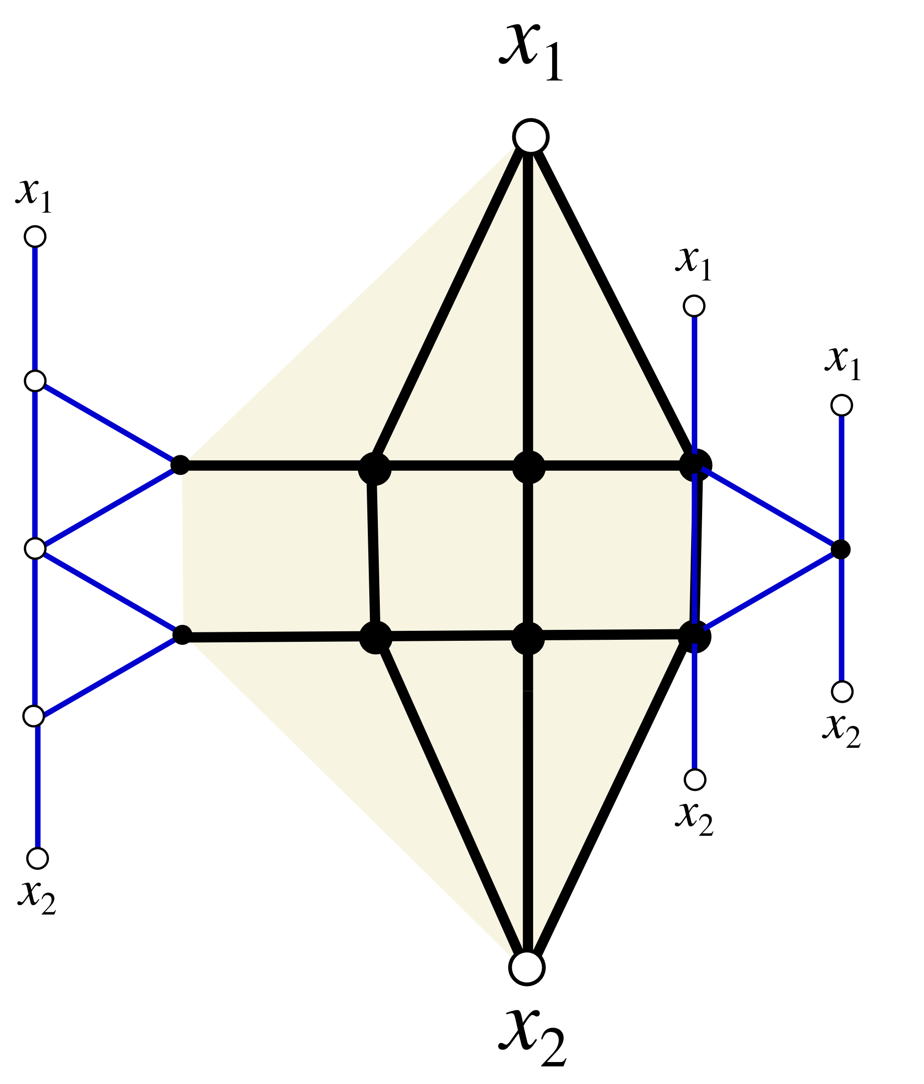




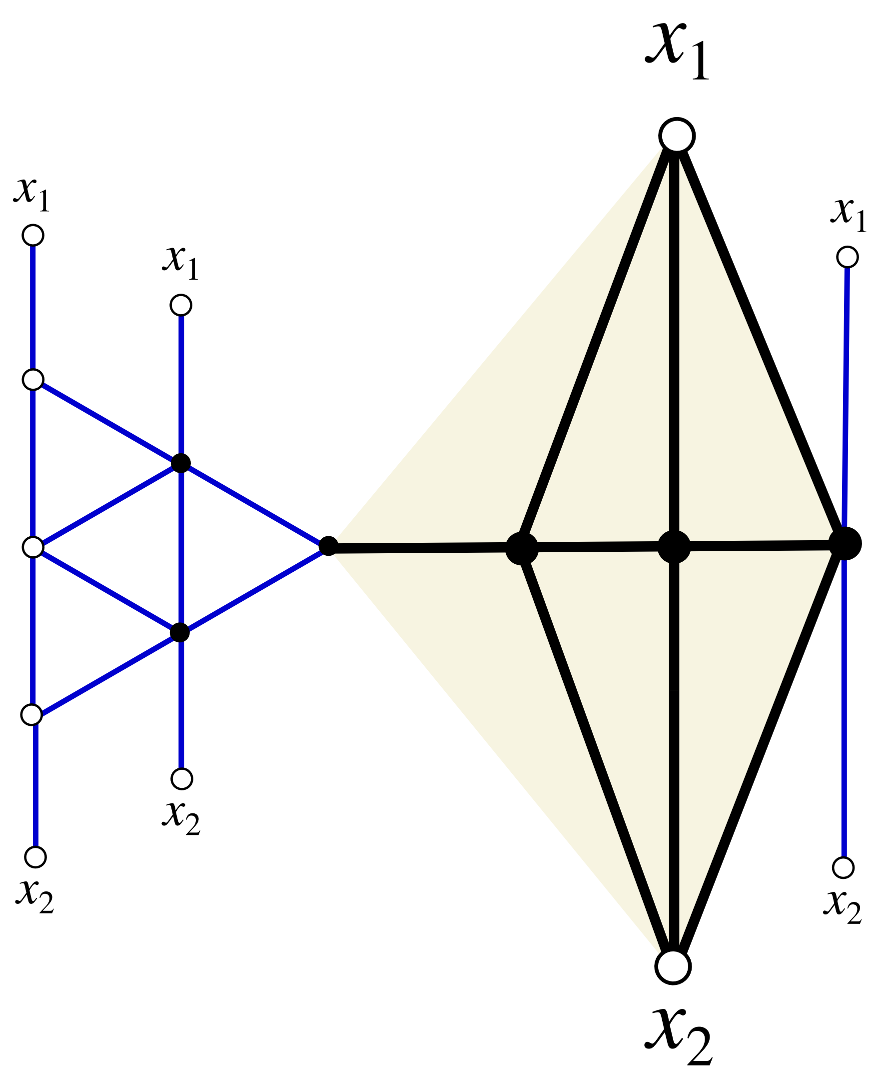
CUTTING



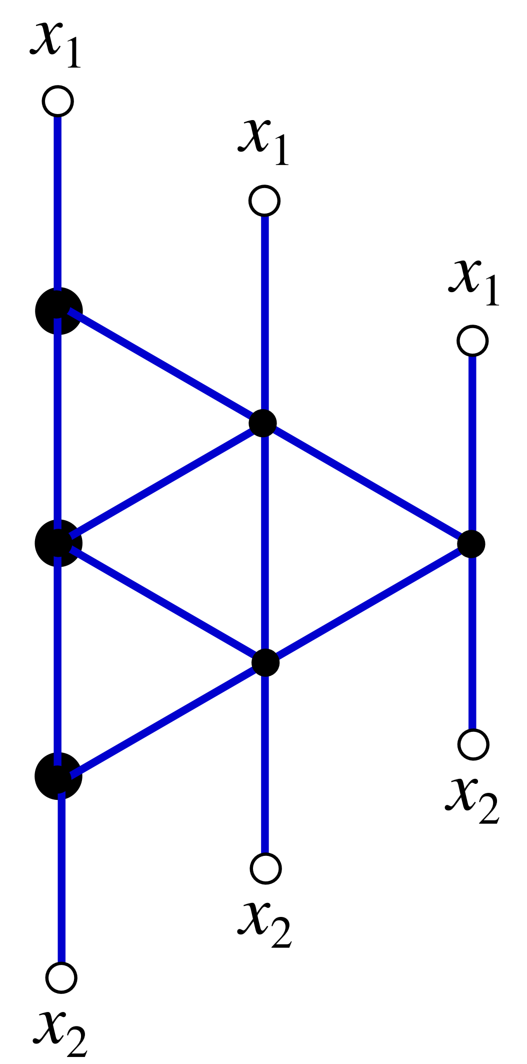




$q(Y_1)^3$

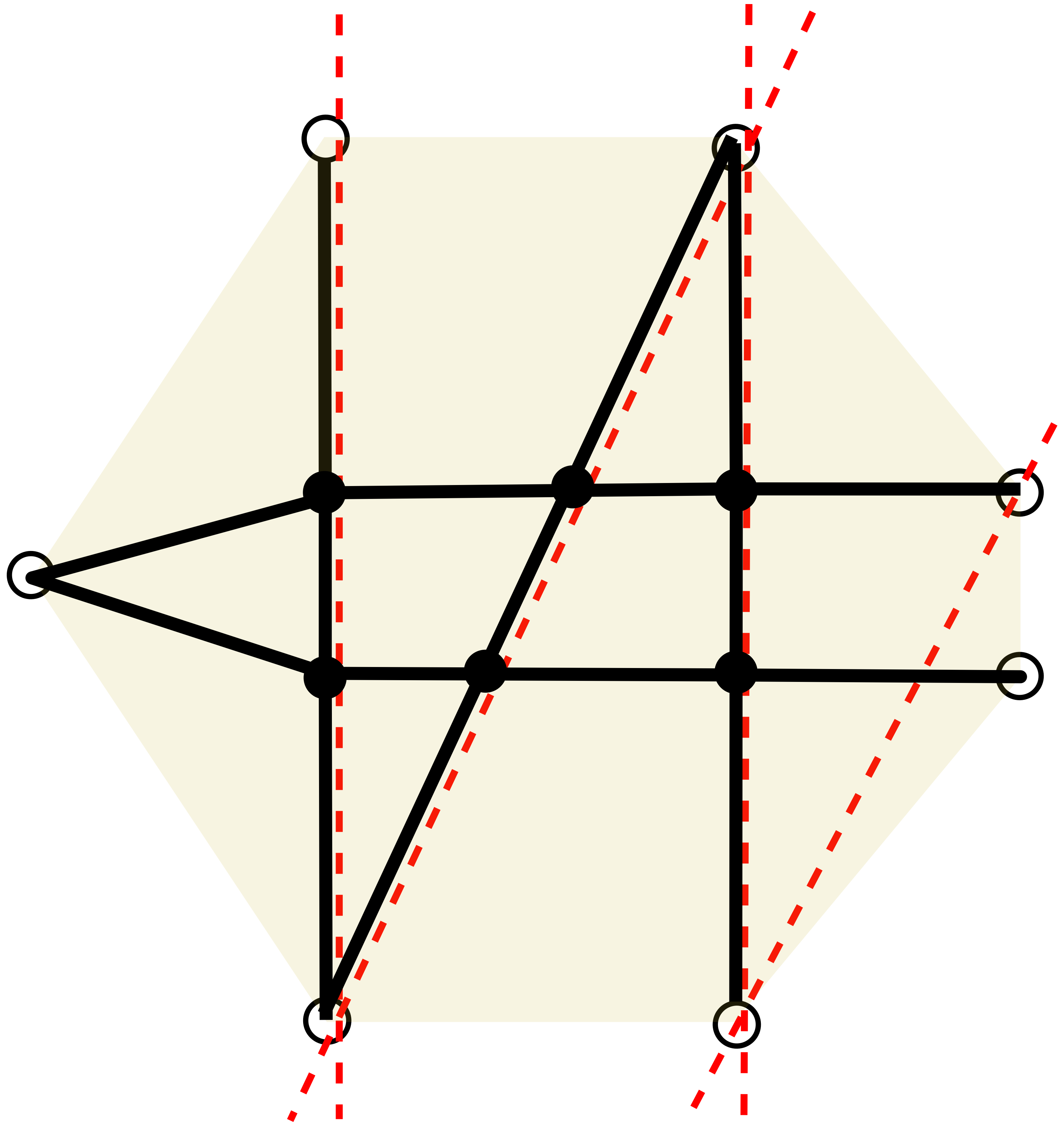


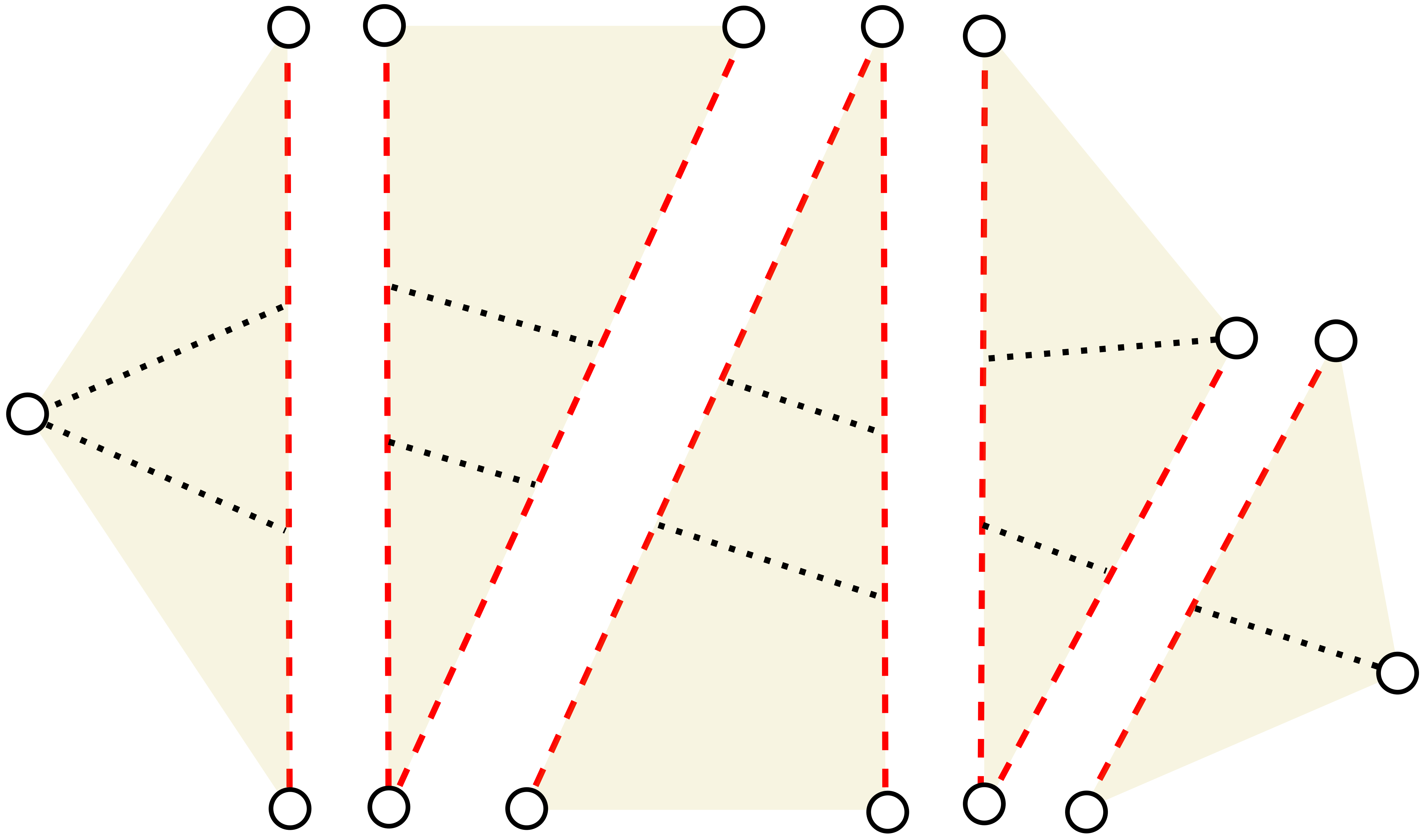
$q(Y_1)^3 q(Y_2)^3$



$q(Y_1)^3 q(Y_2)^3 q(Y_3)^3$

OVERLAPPING





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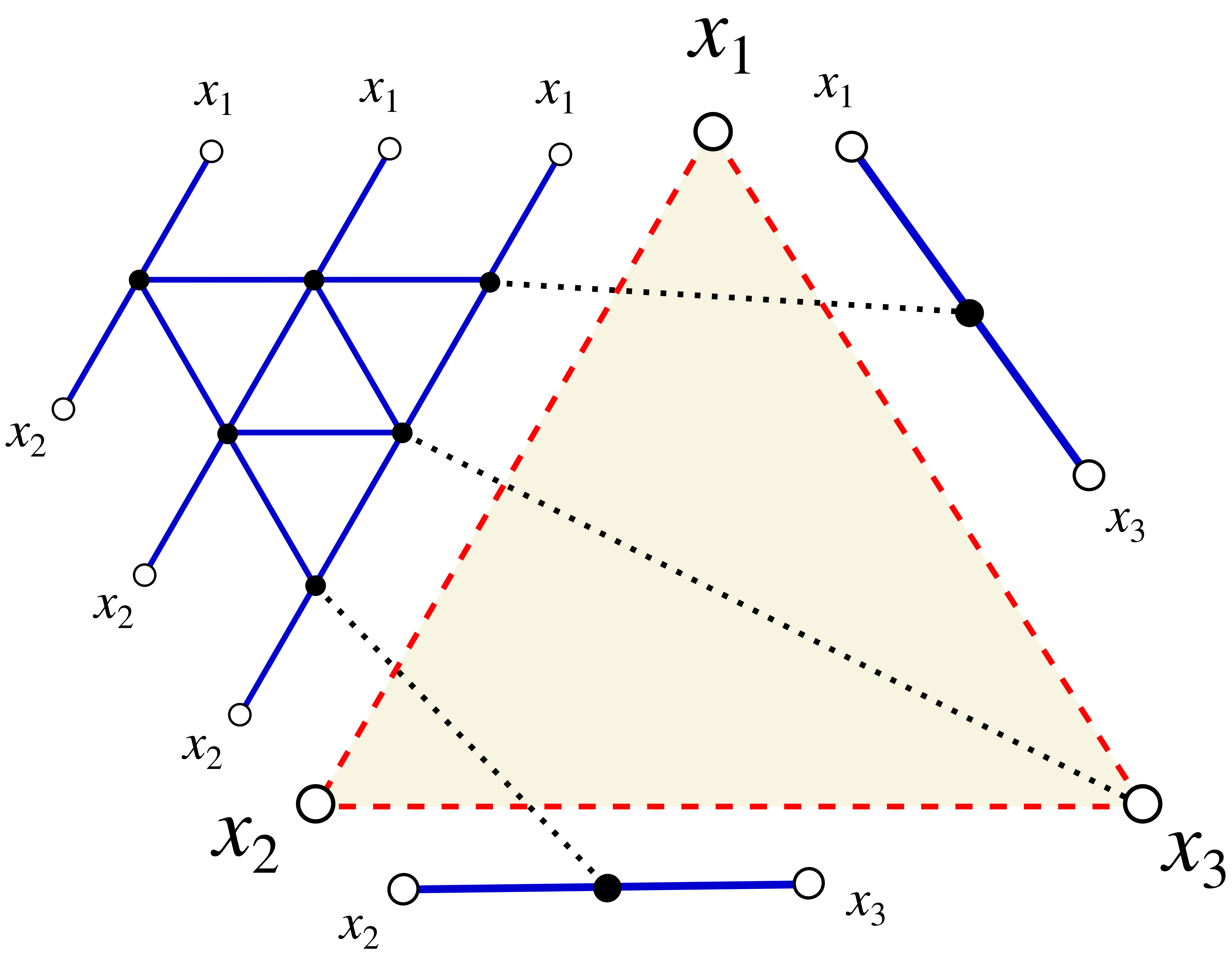

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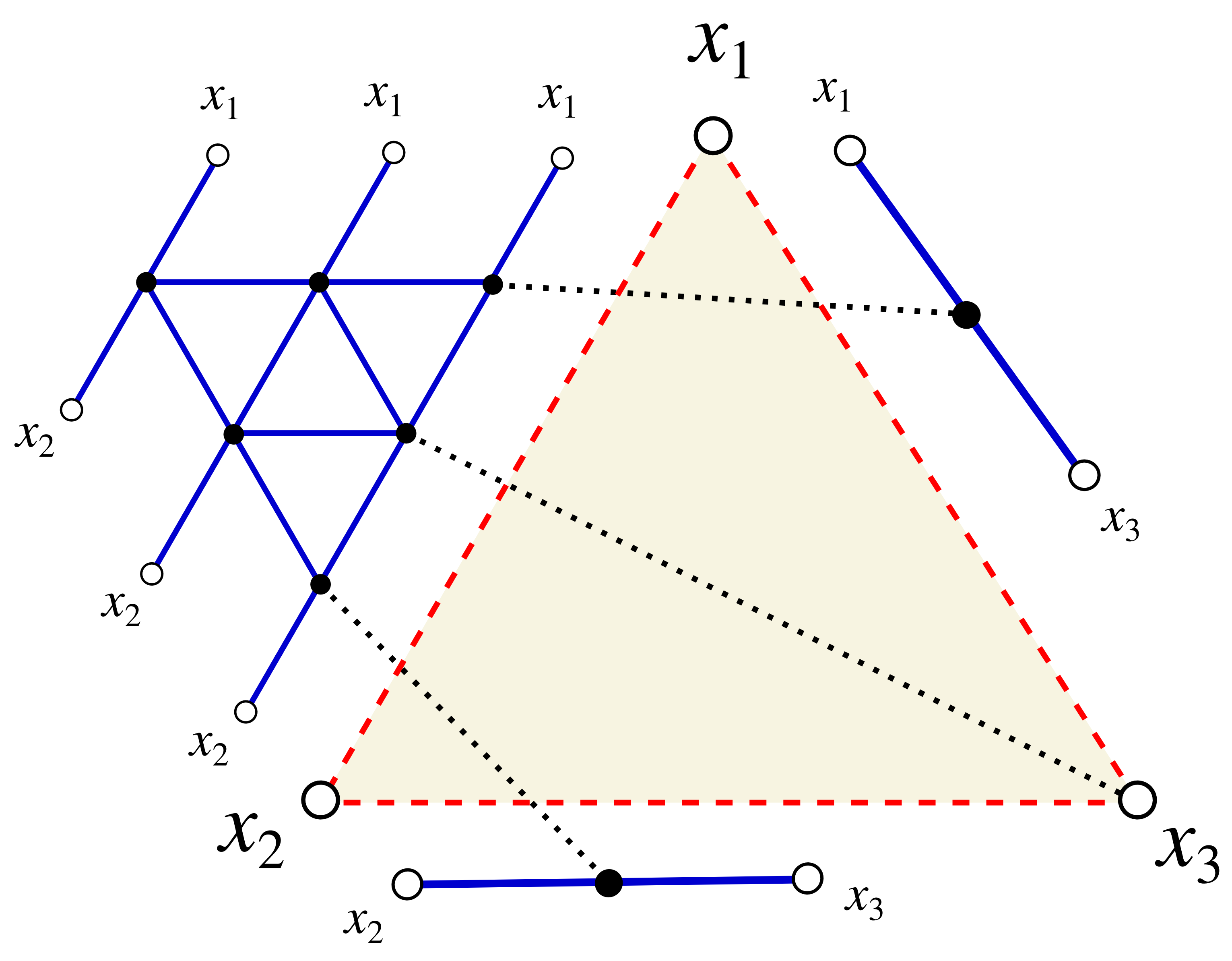

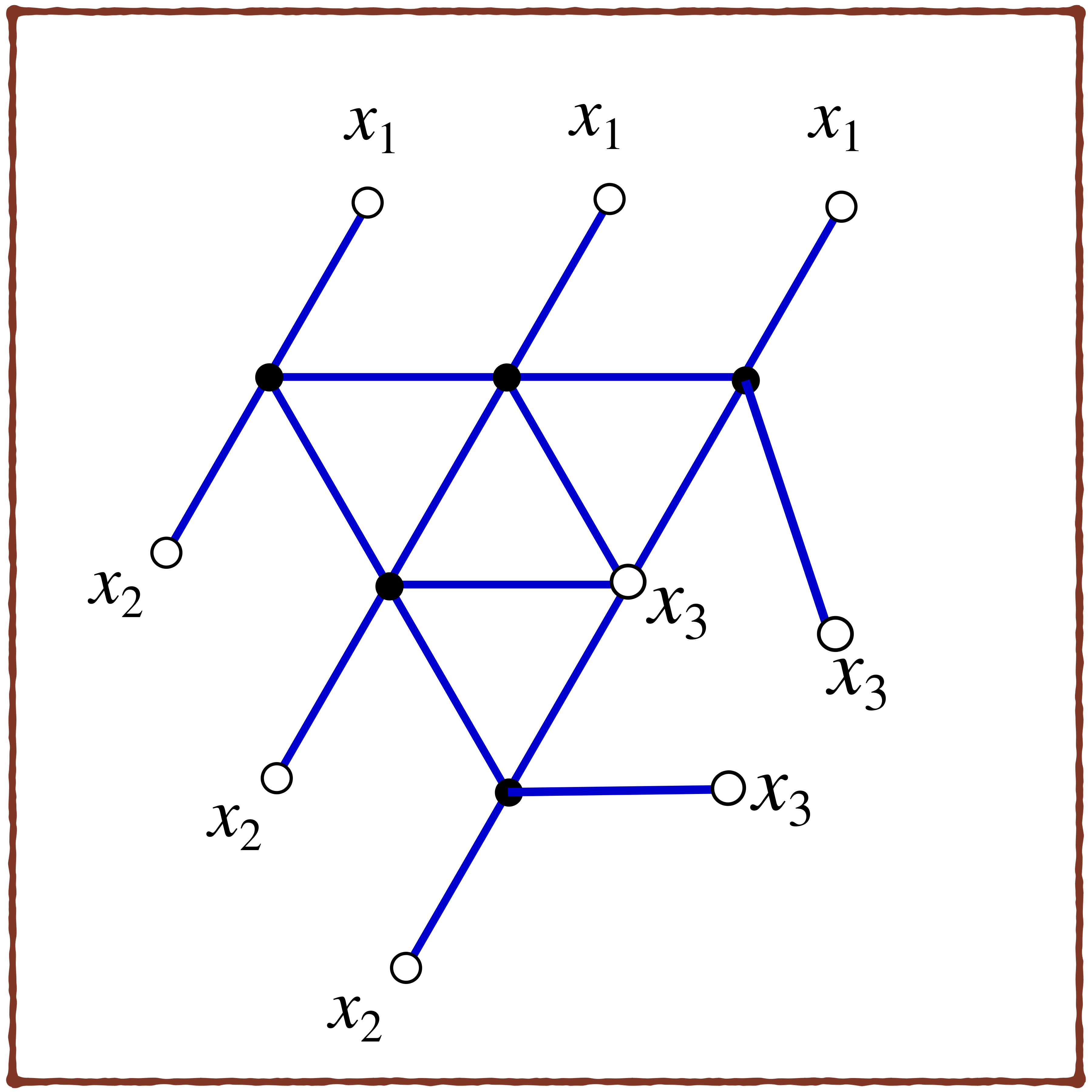

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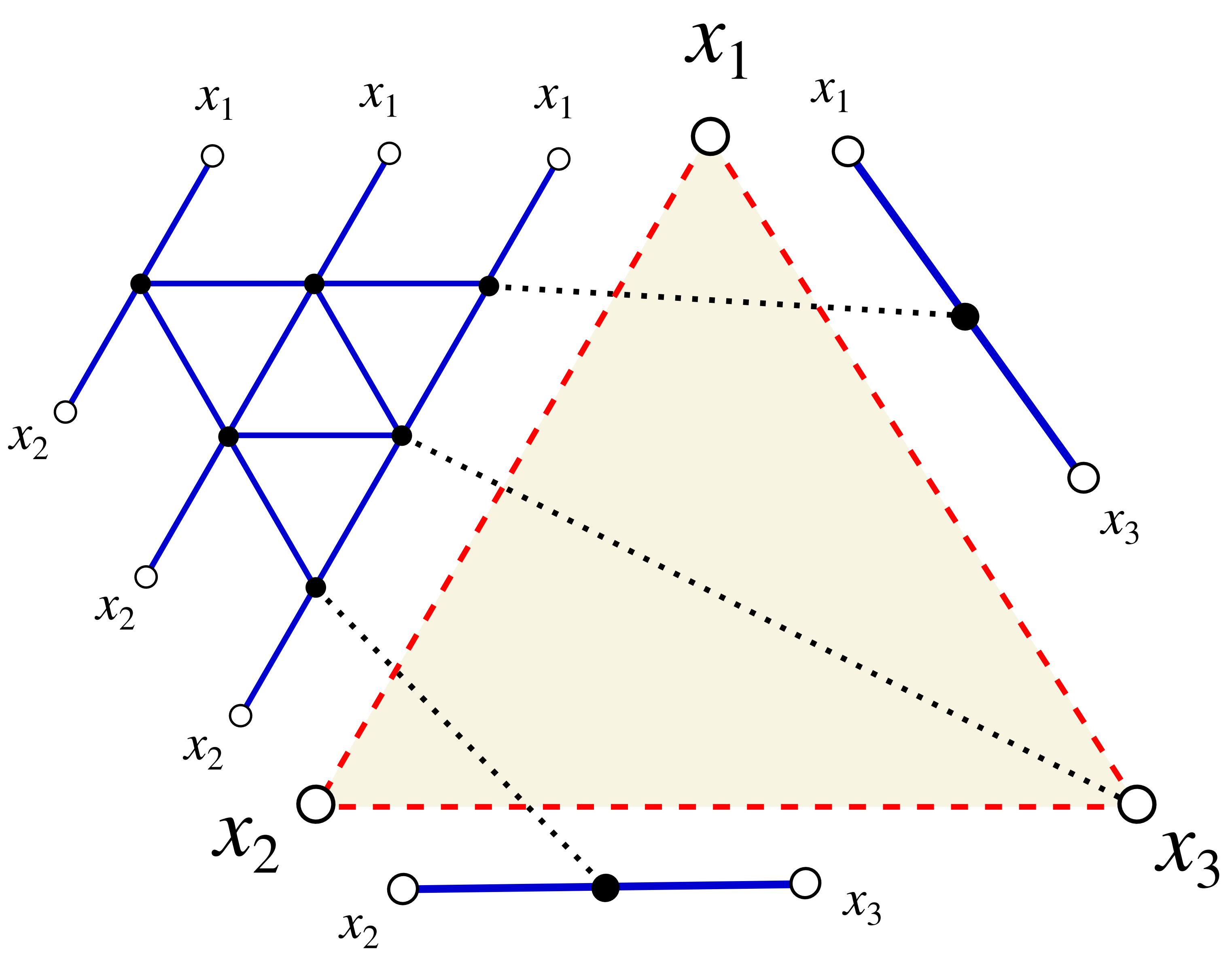

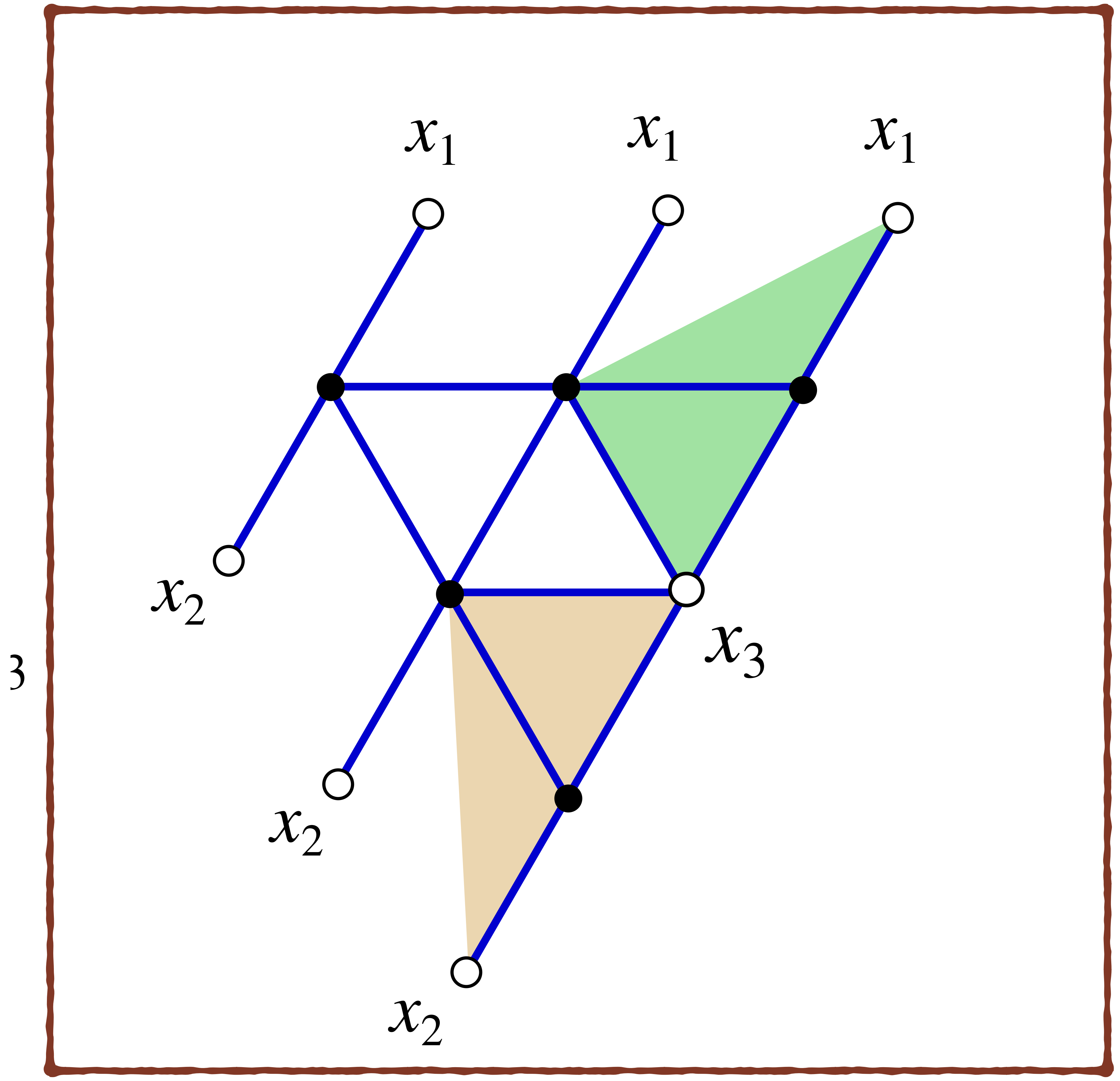

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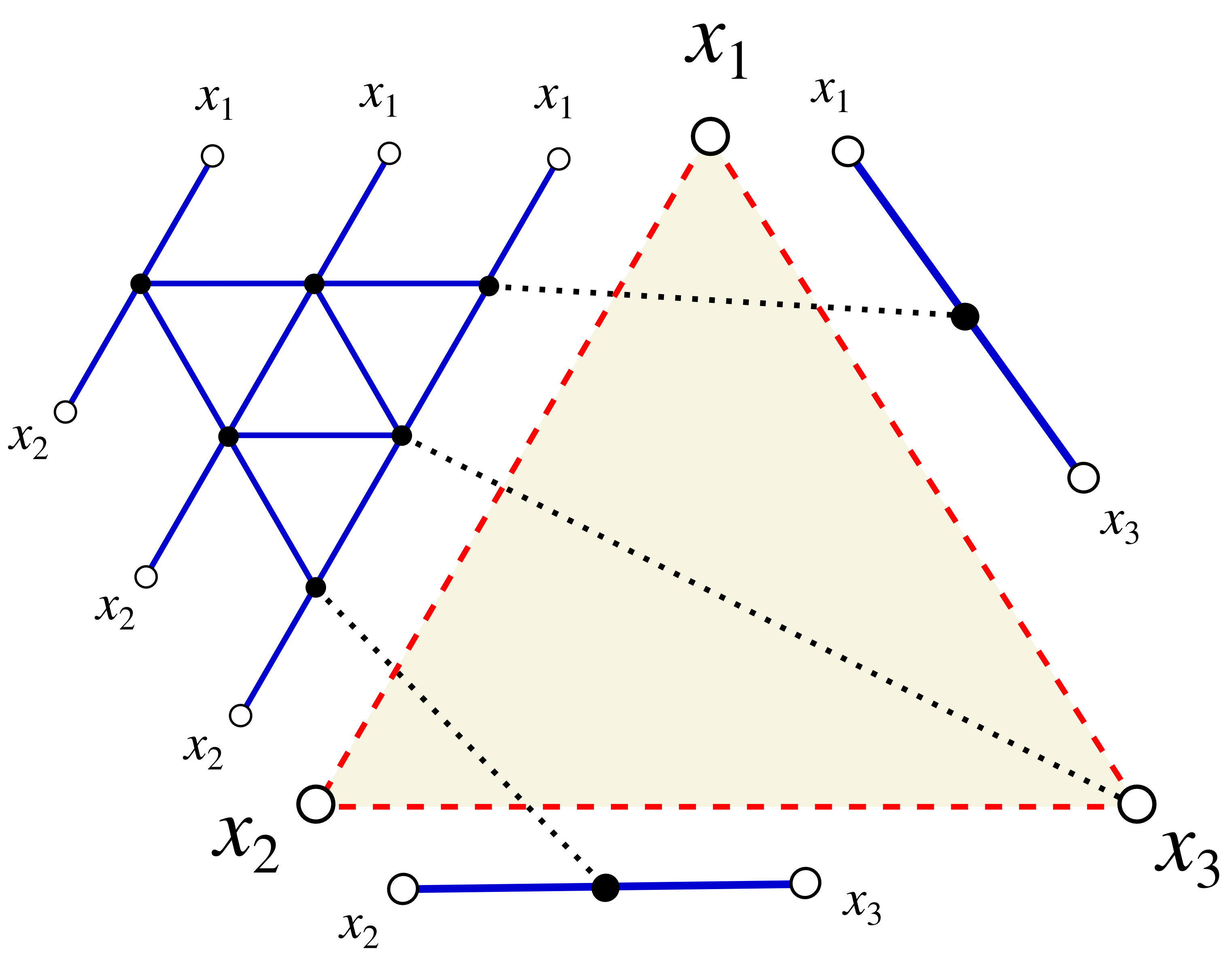

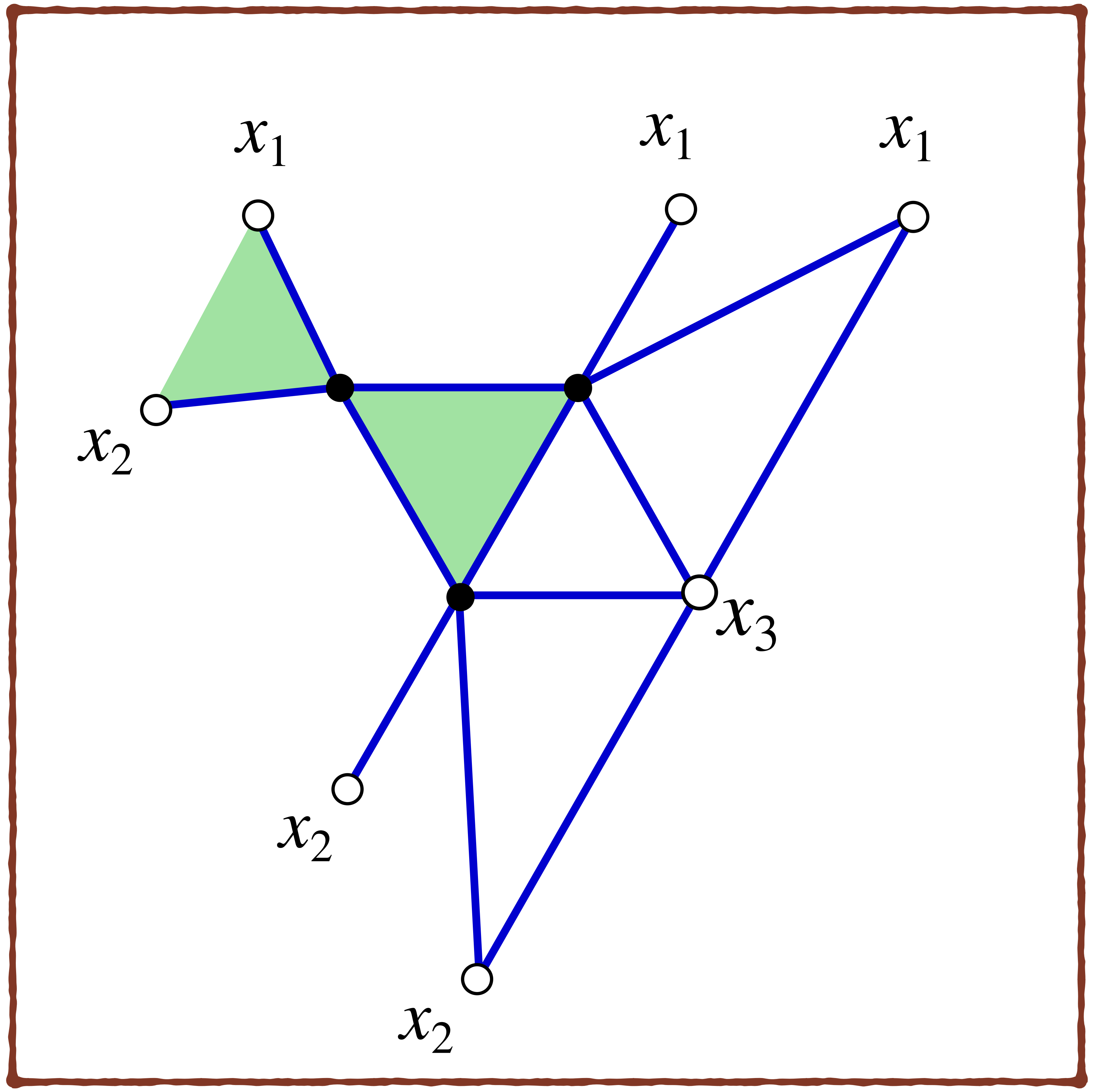

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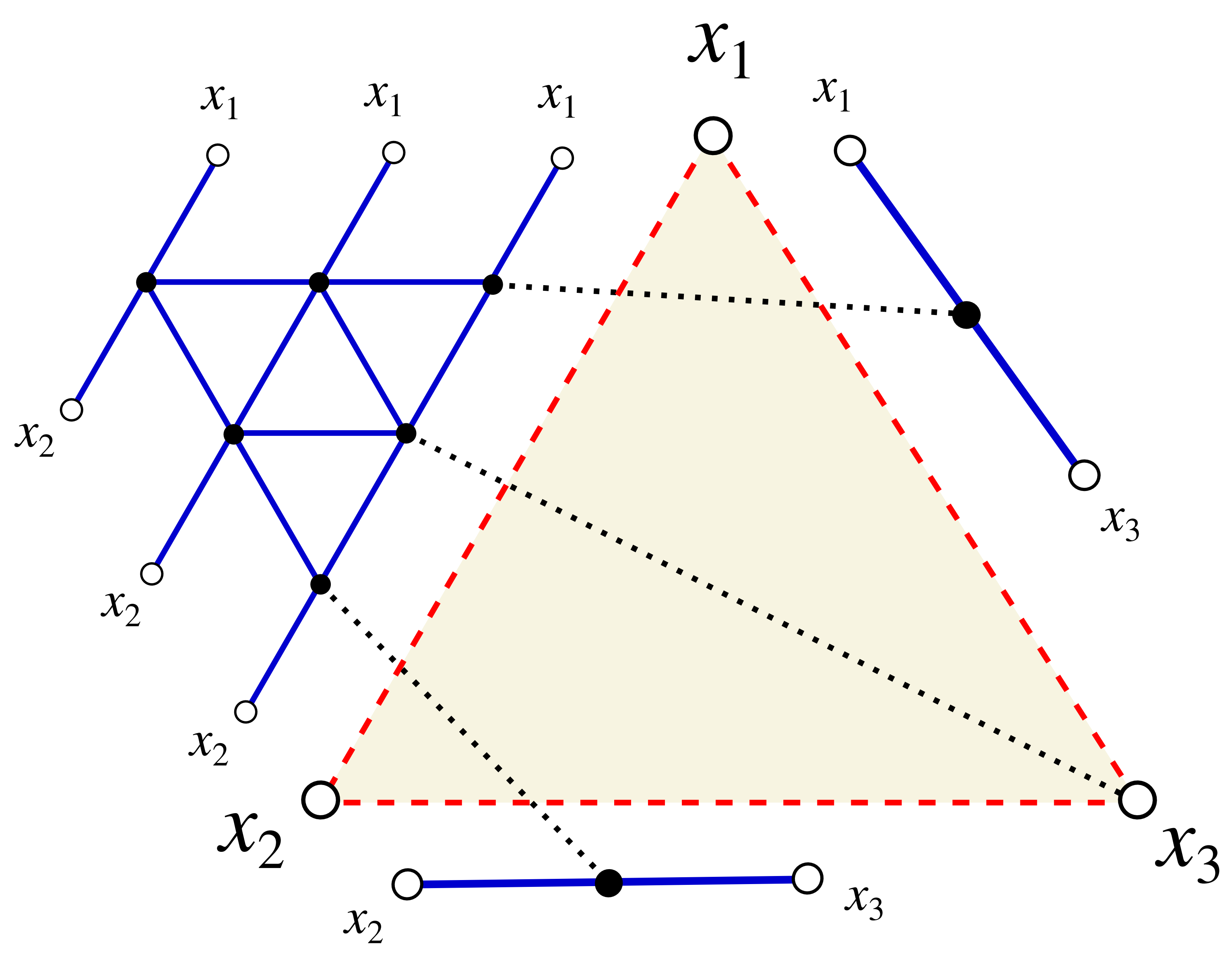

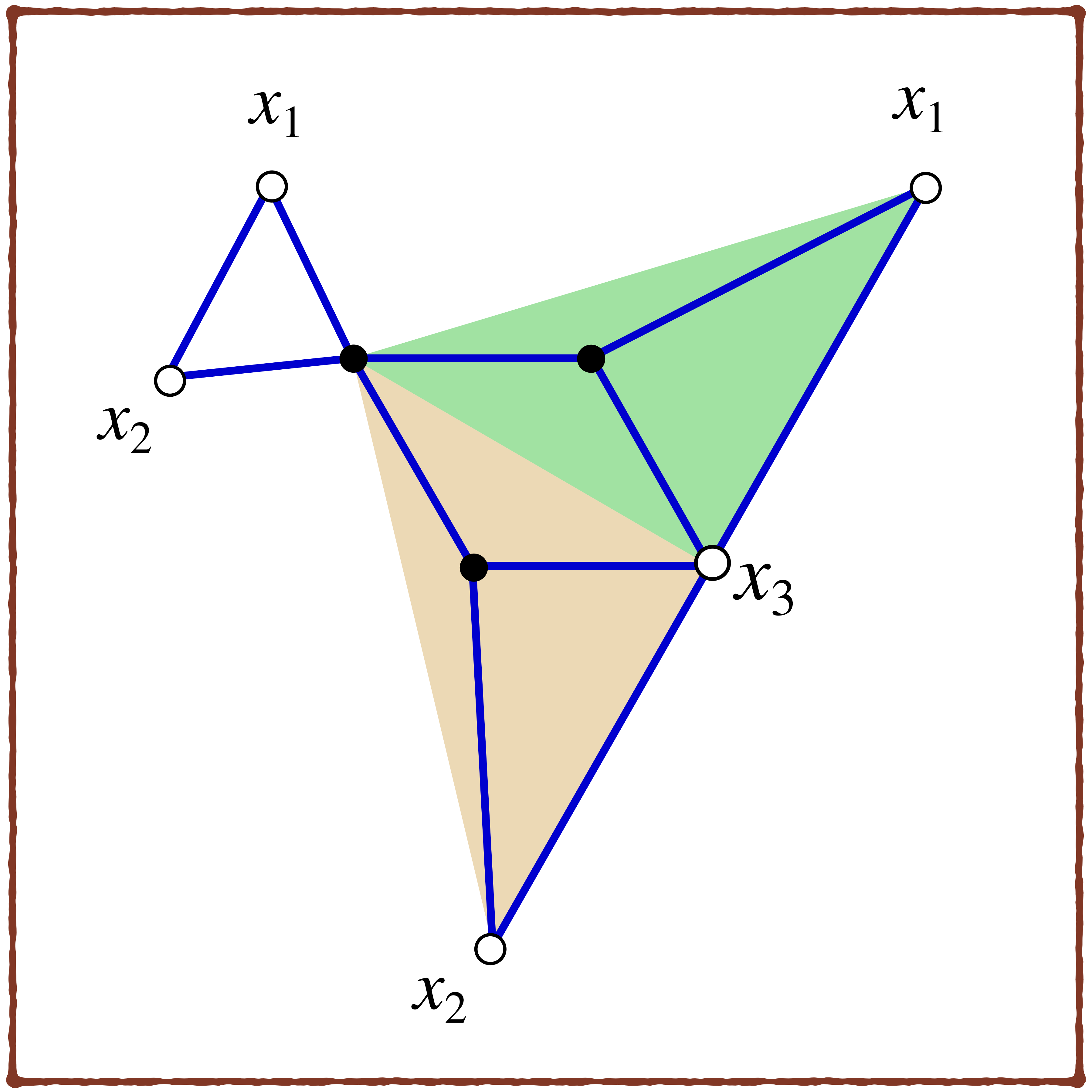

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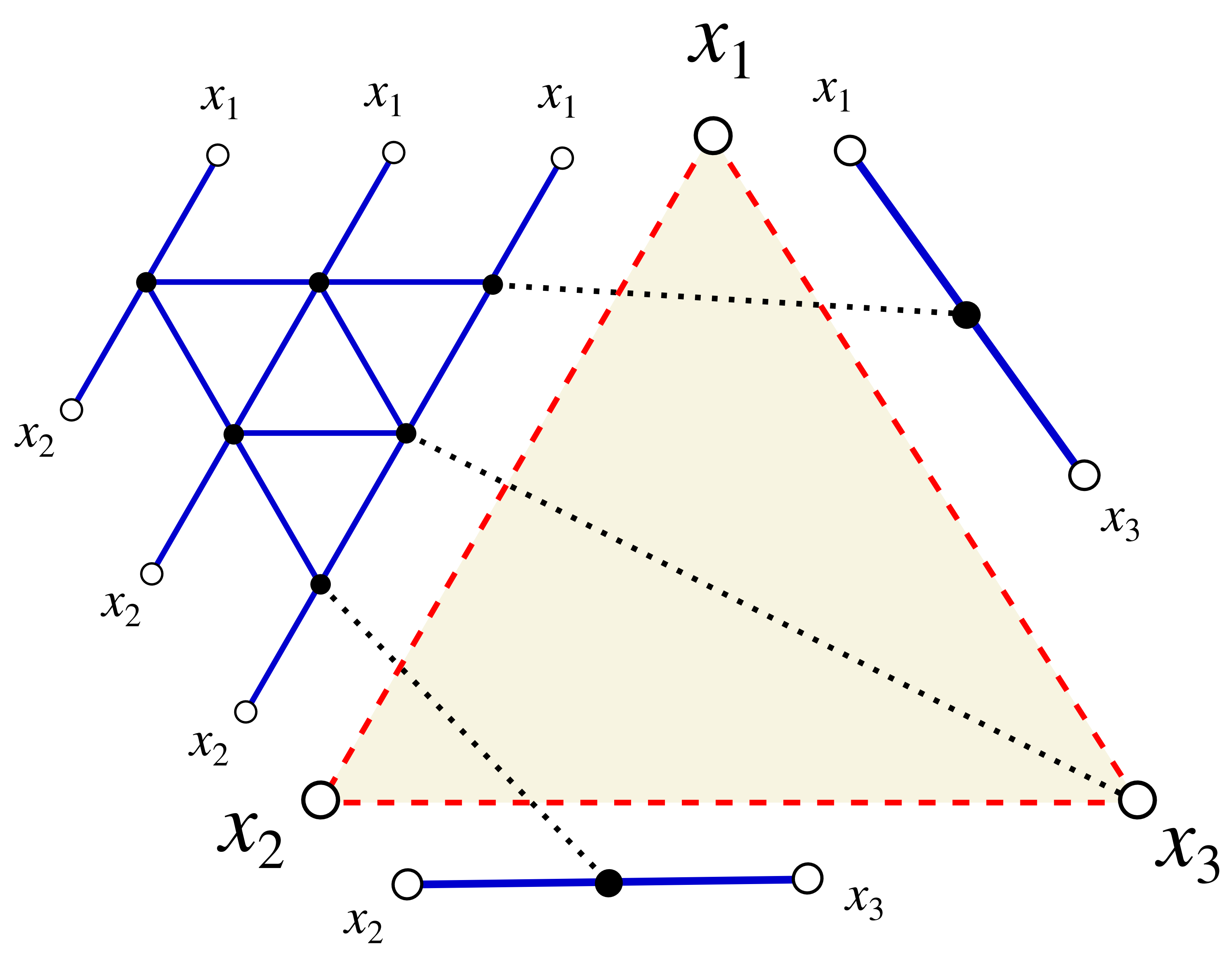

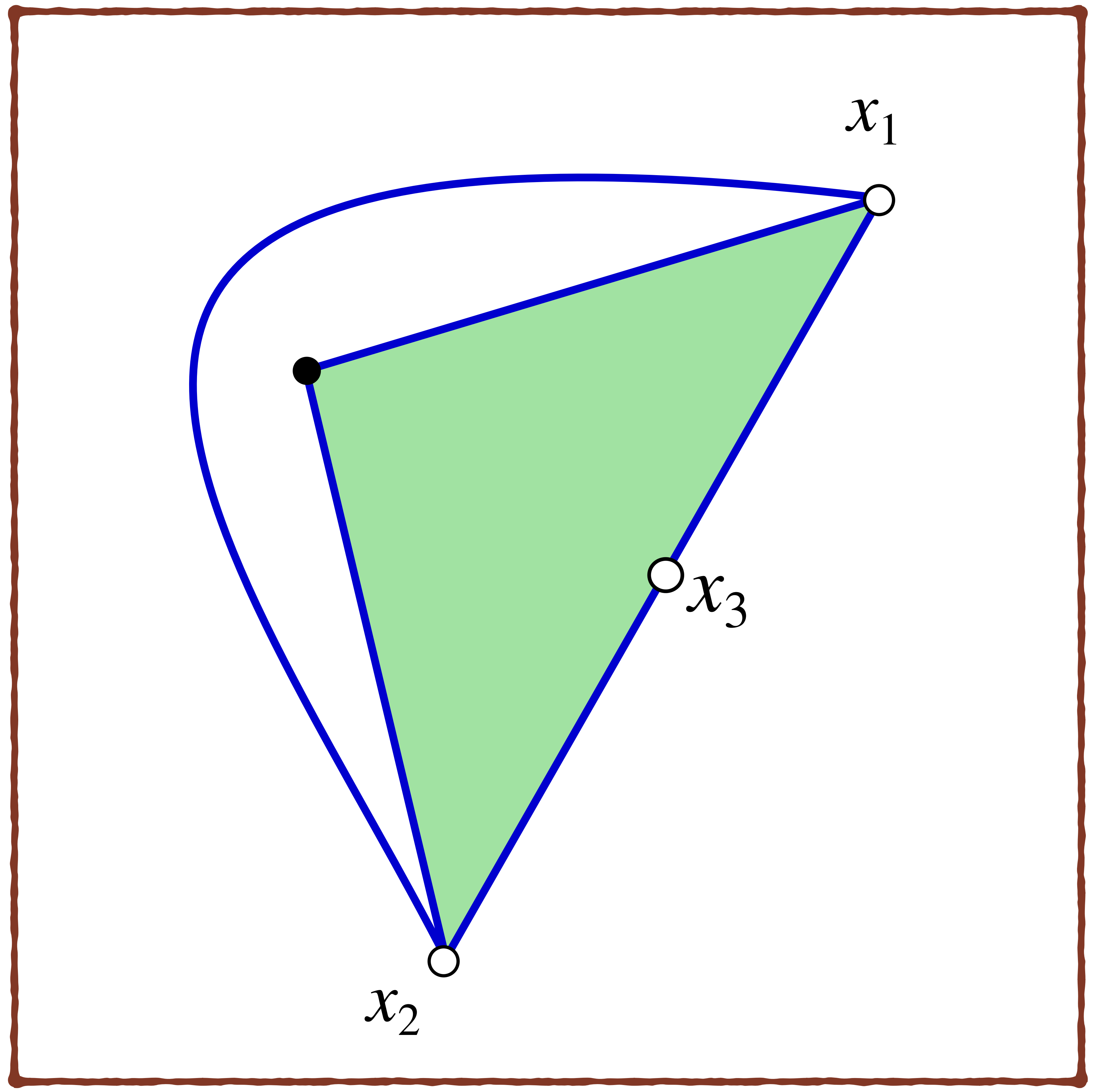

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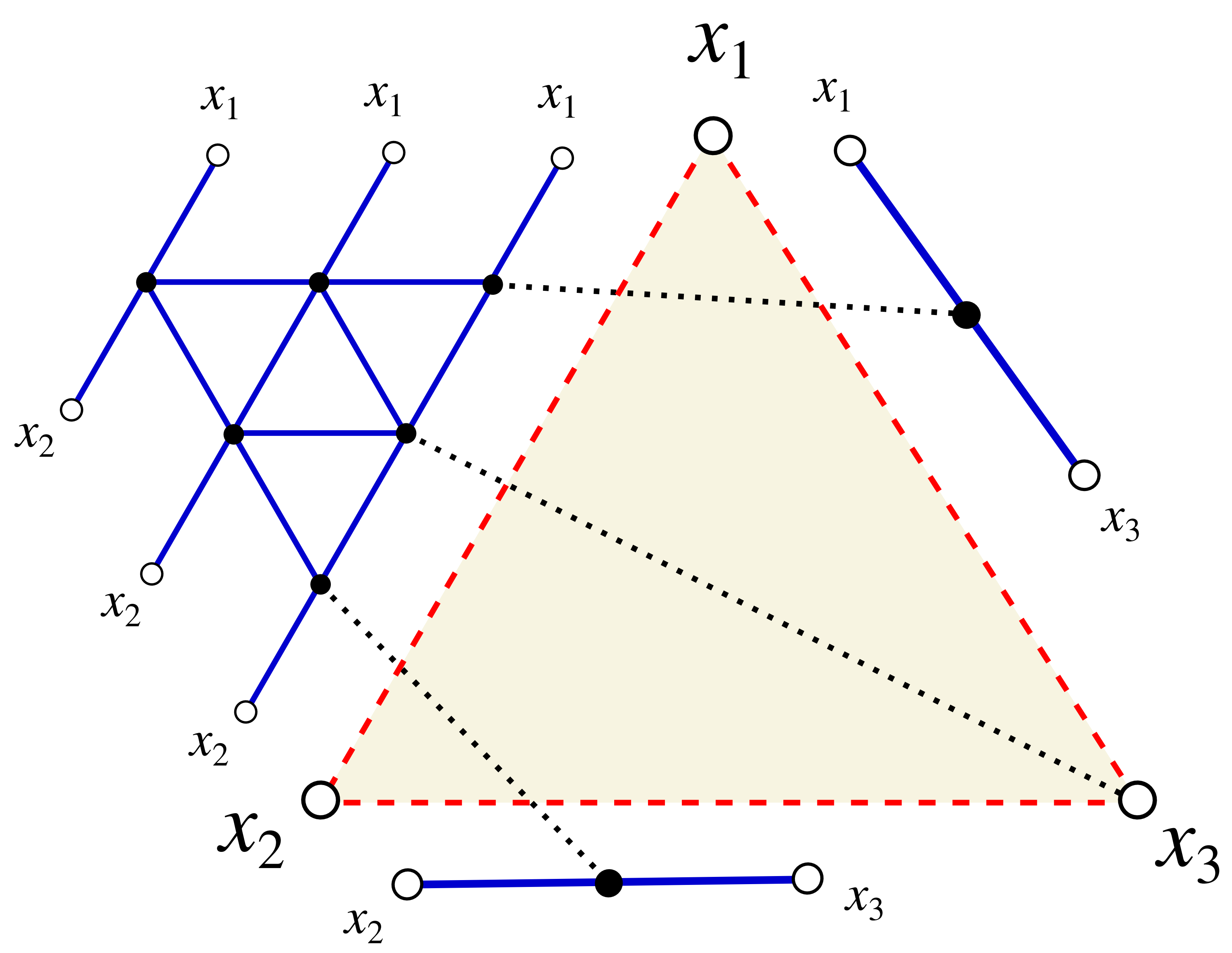
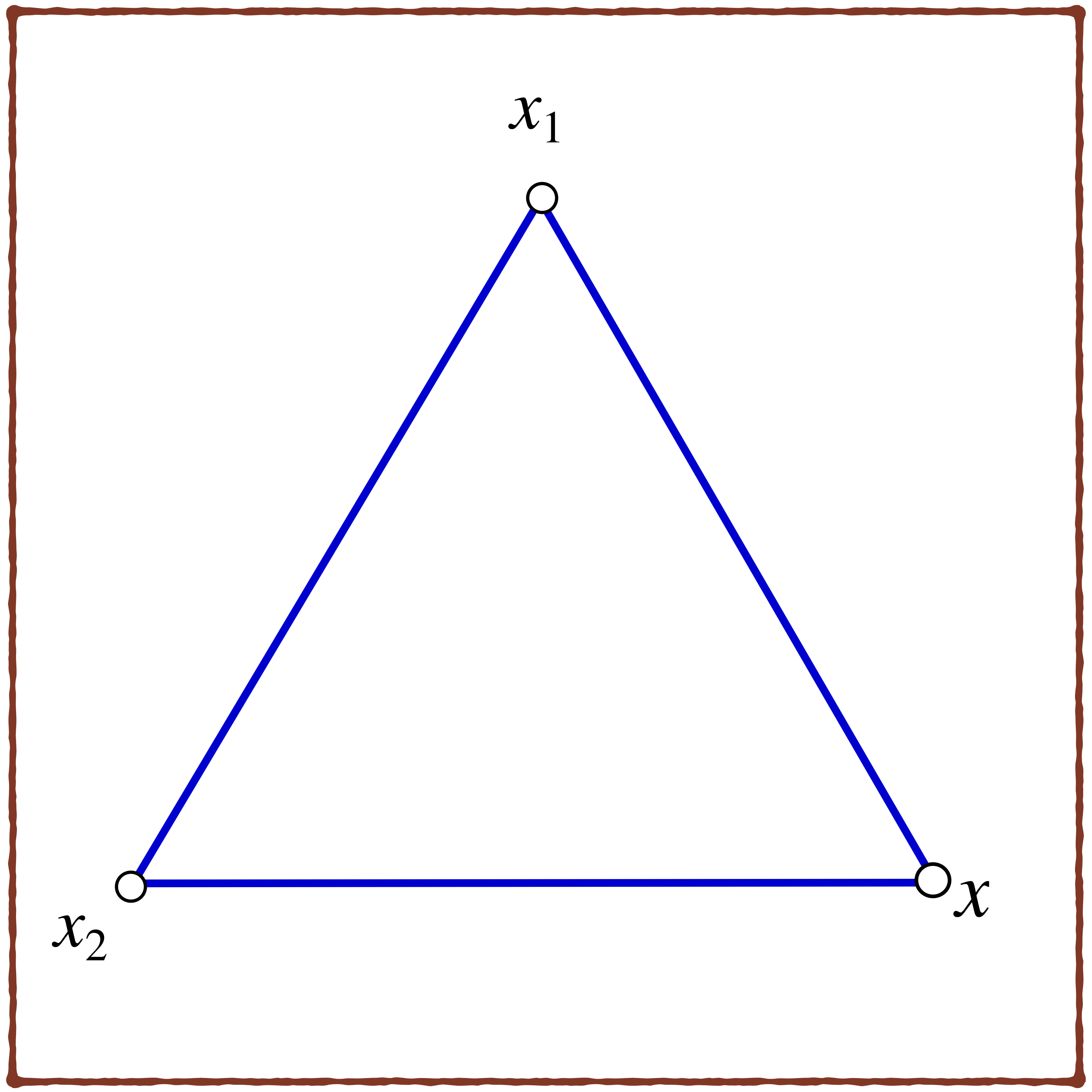
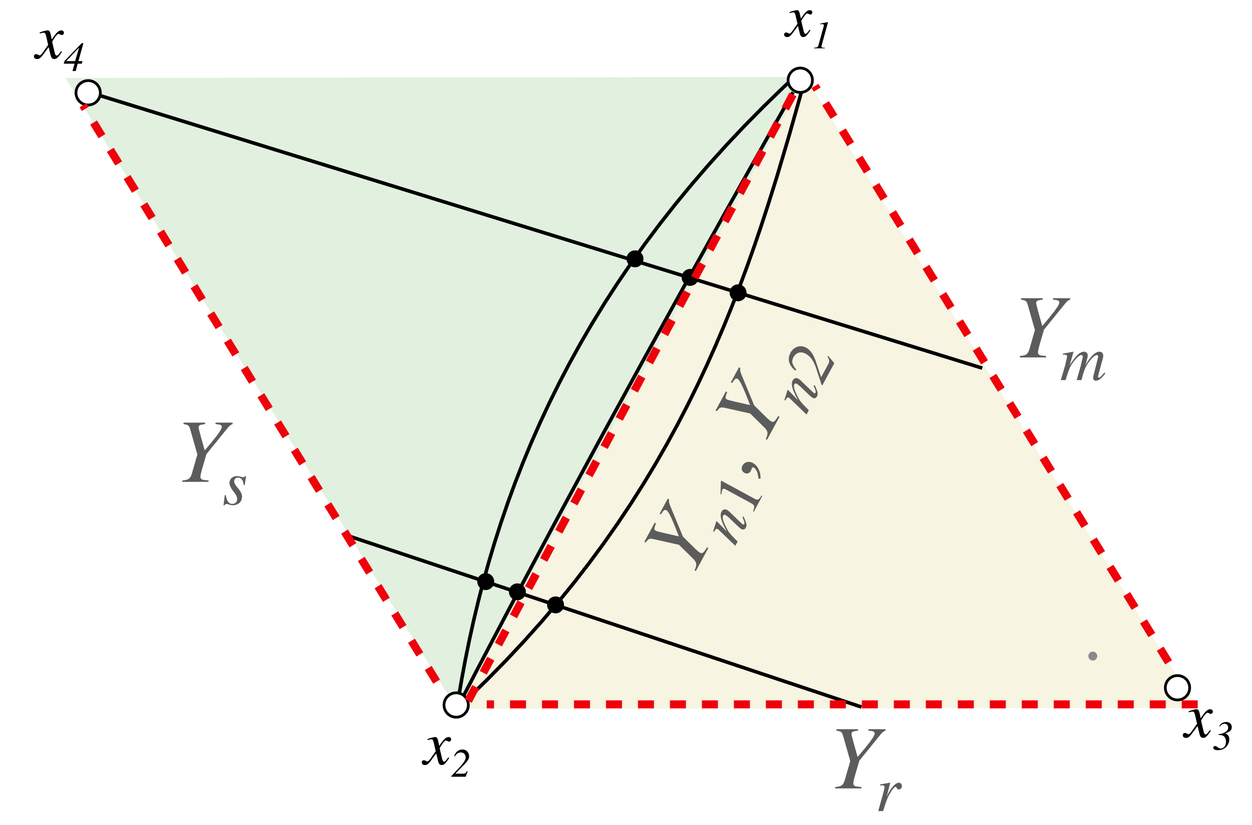


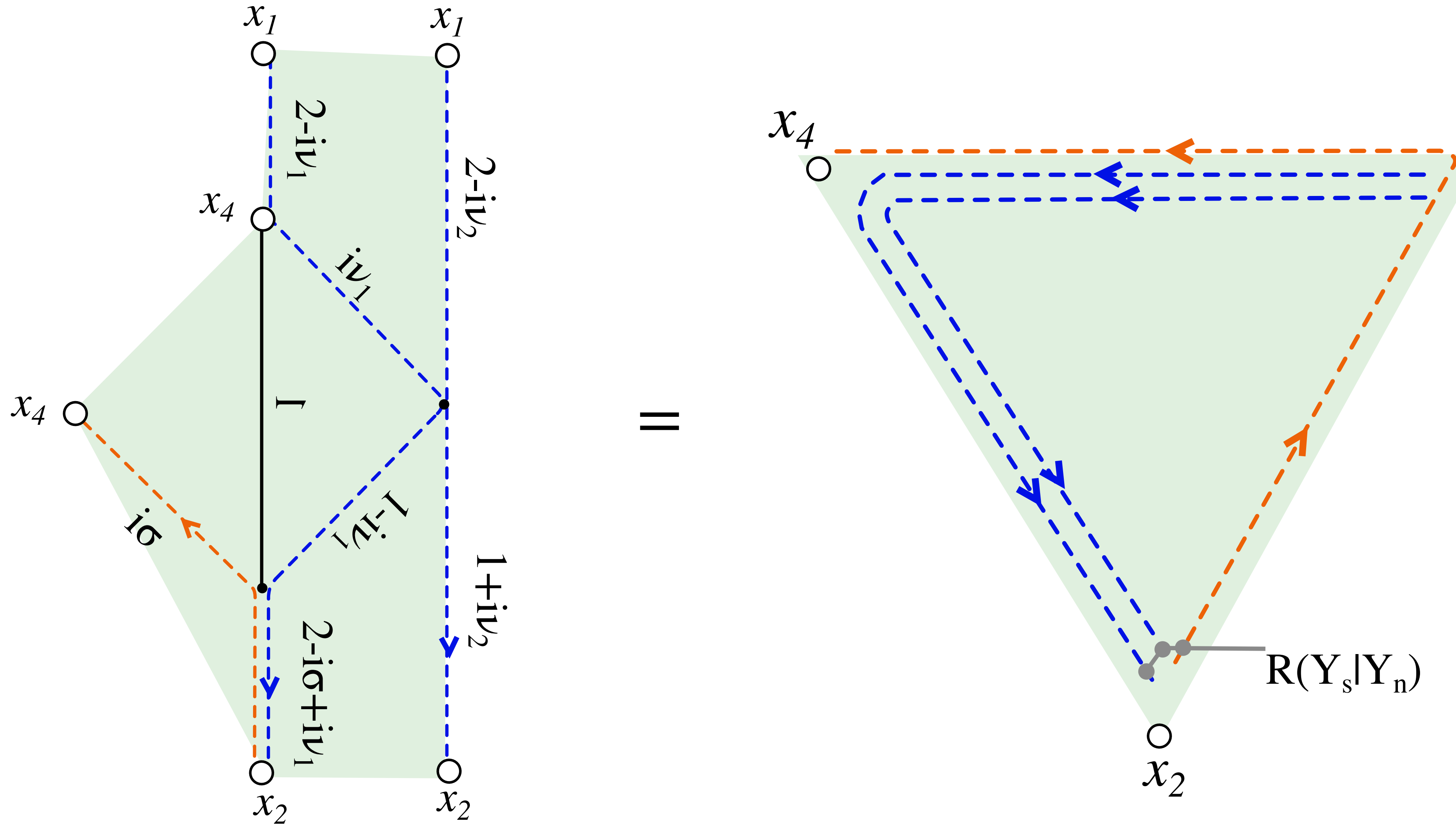


ALL SPACETIME INTEGRATIONS ARE GONE!

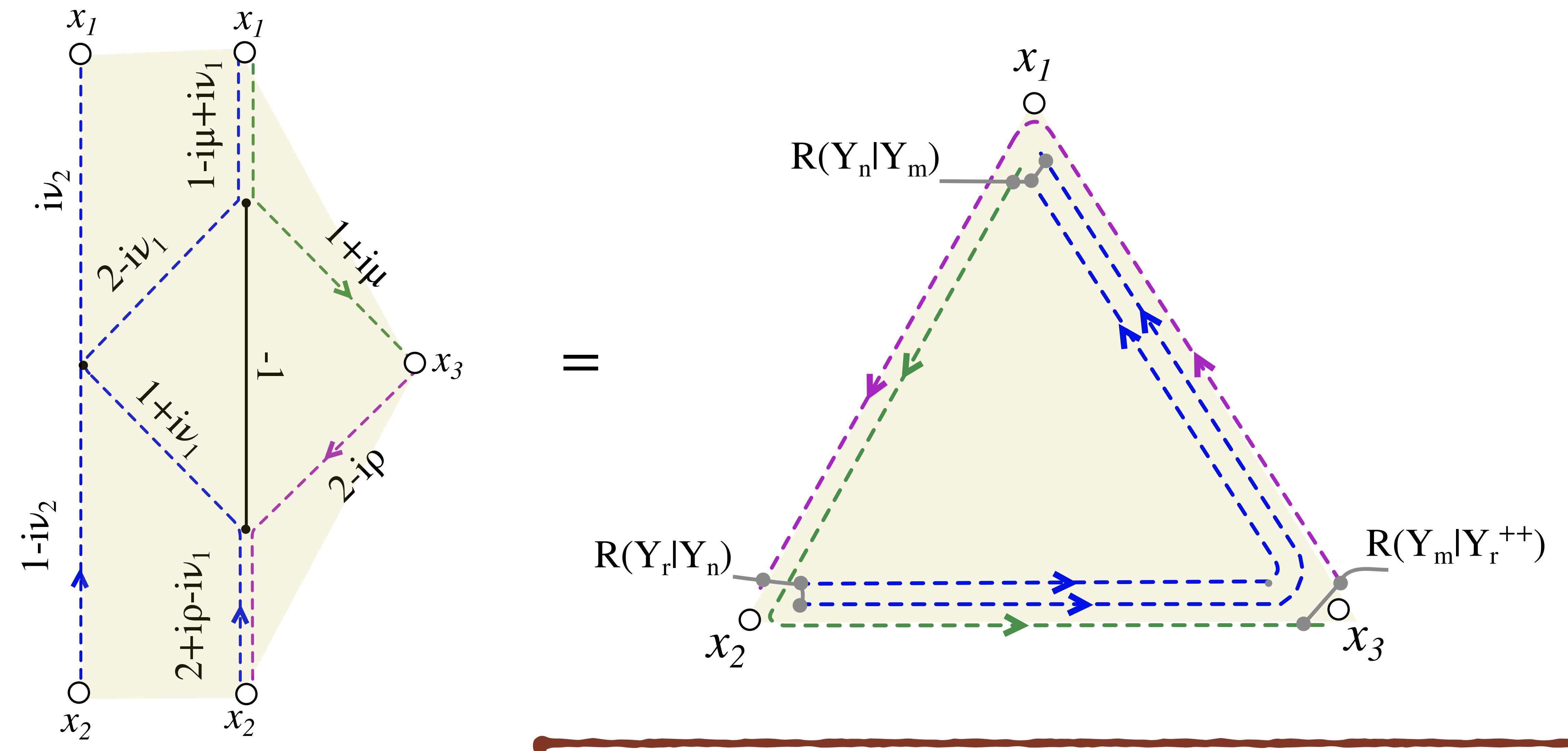
GLUING TWO TRIANGLES



$$H(Y_s \,|\, Y_n) = \frac{\Gamma\left(\frac{n}{2} + i\nu\right)\Gamma\left(\frac{s-n}{2} - i\nu + i\sigma\right)}{\Gamma\left(1 + \frac{s}{2} - i\sigma\right)\Gamma\left(1 + \frac{n}{2} - i\nu\right)\Gamma\left(1 + \frac{s-n}{2} + i\nu - i\sigma\right)}$$

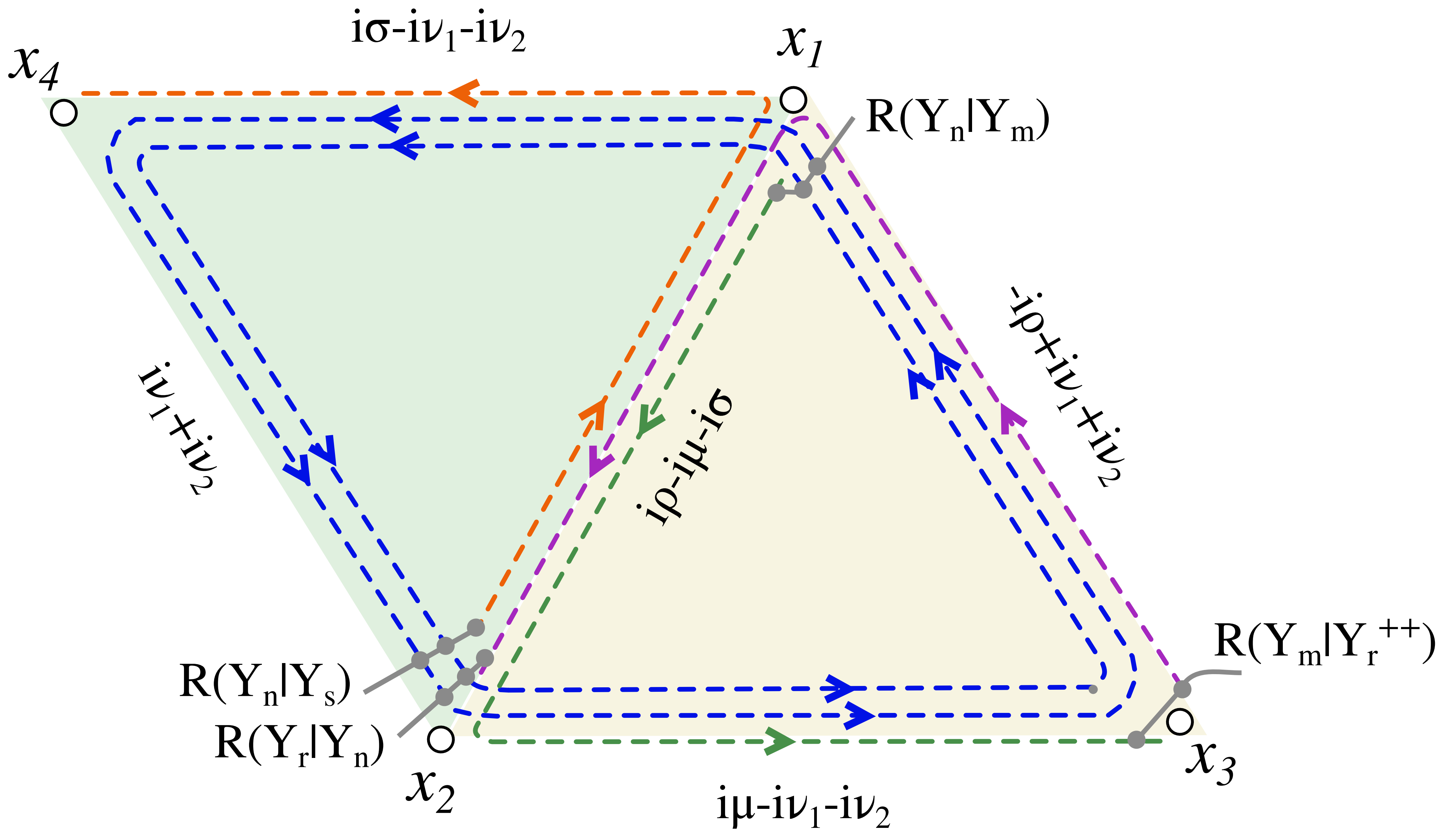


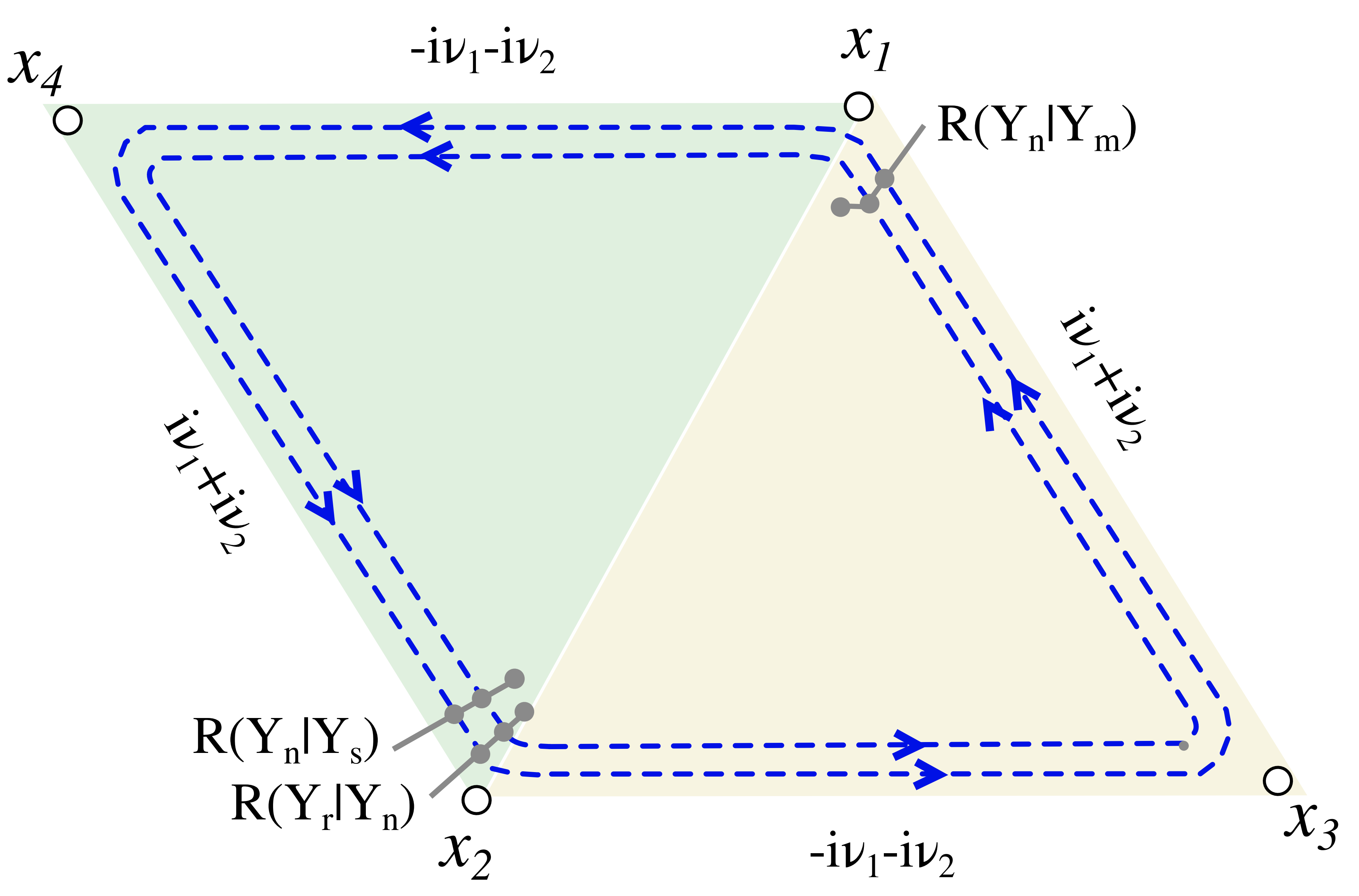
$$\mathbb{H}(Y_s \,|\, Y_n) = H(Y_s \,|\, Y_n)$$



$$\mathbb{H}(Y_n \,|\, Y_m, Y_r) = \frac{H(Y_n \,|\, Y_m) H(Y_n \,|\, Y_r)}{H(Y_r \,|\, Y_m)}$$

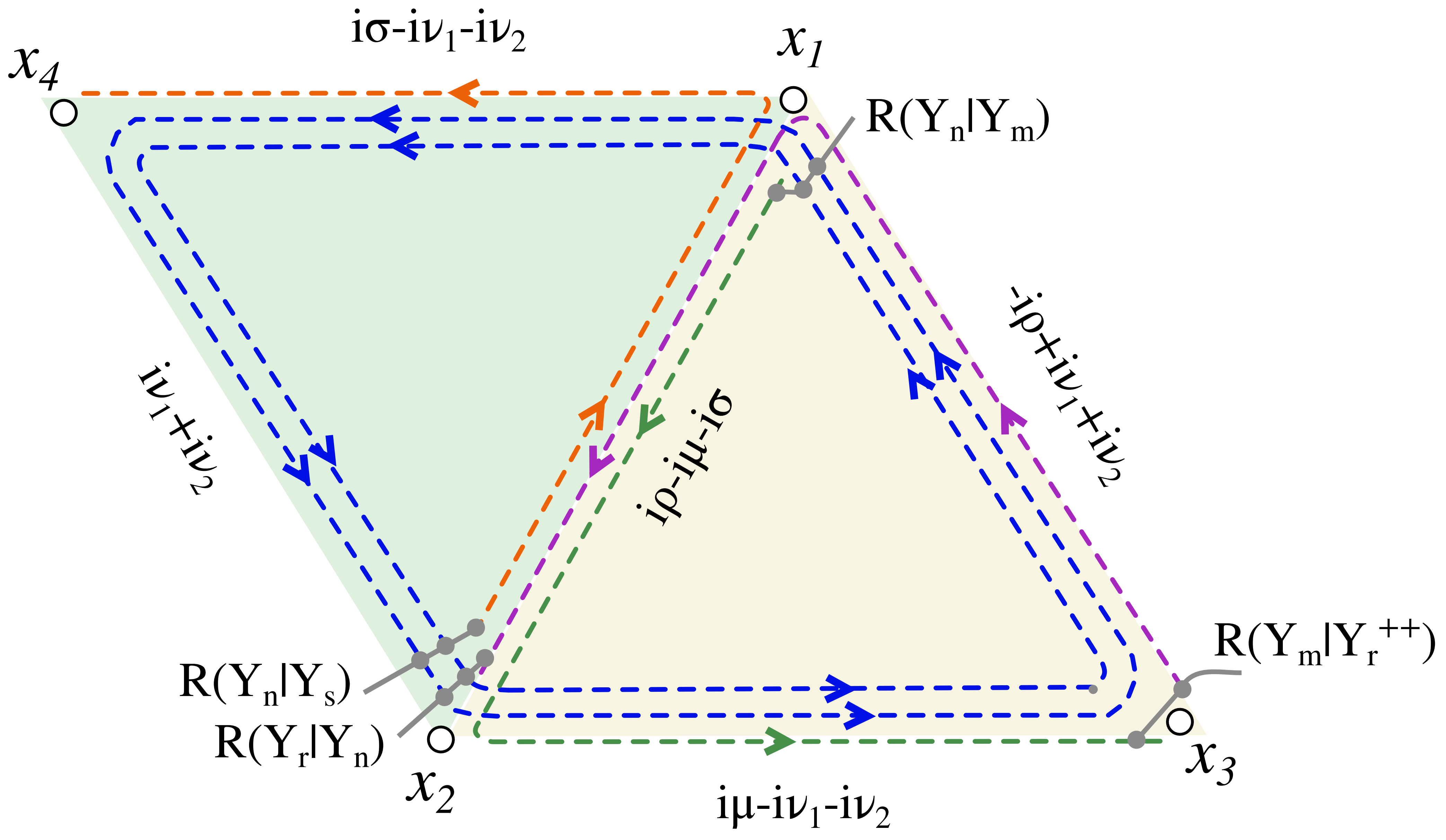
GLUING TWO TRIANGLES

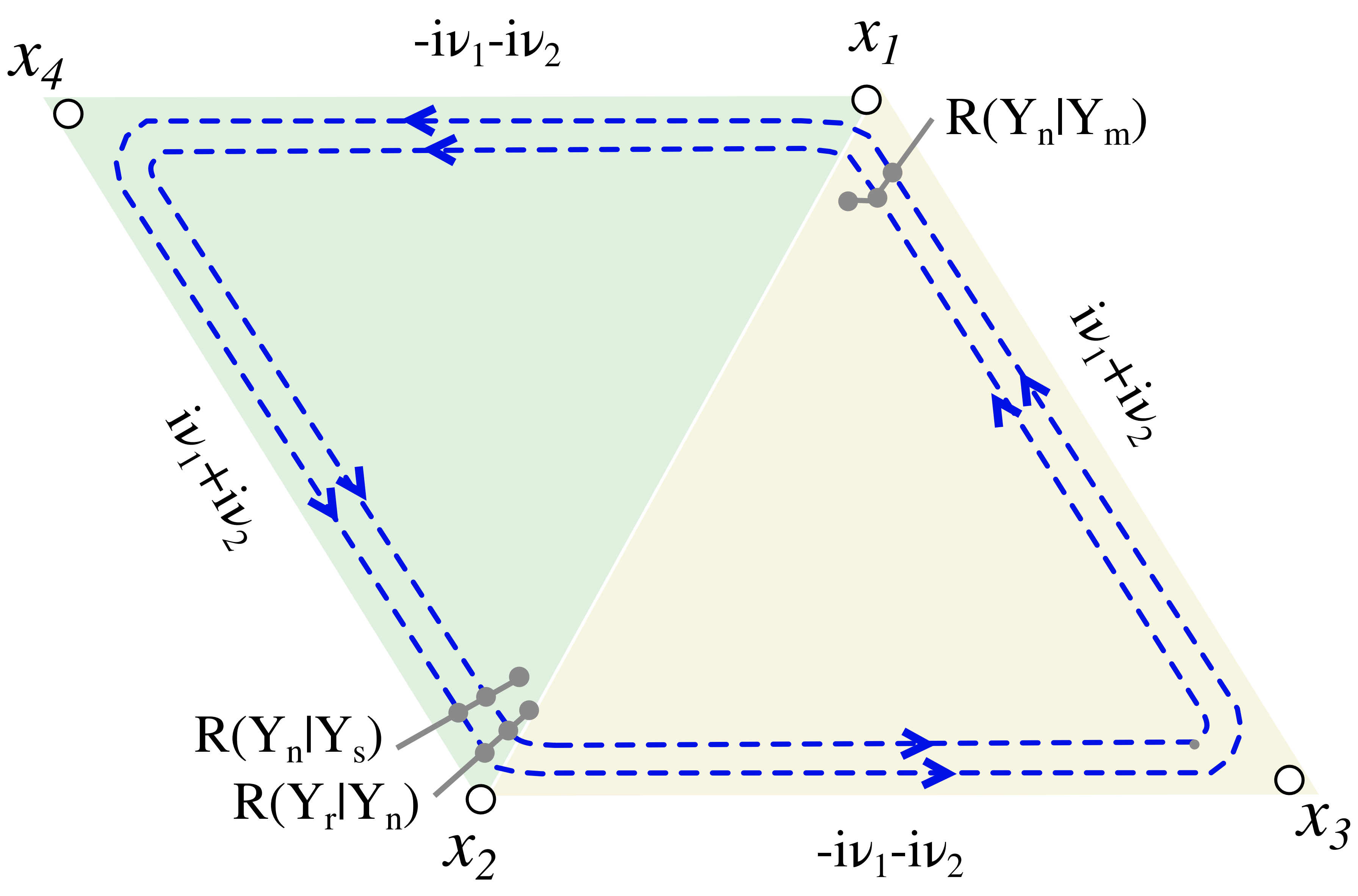




$$W(x_j, Y_{n_k}) = \left(\frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}\right)^{i\nu_k} \times Tr_{n_k}\left[\left[\mathbf{x_{23}\overline{x_{31}}}\right]^{n_k}\mathbf{R}(Y_{n_k}\,|\,Y_m)\left[\mathbf{x_{14}\overline{x_{43}}}\right]^{n_k}\mathbf{R}(Y_{n_k}\,|\,Y_s)\mathbf{R}(Y_r\,|\,Y_{n_k})\right]$$

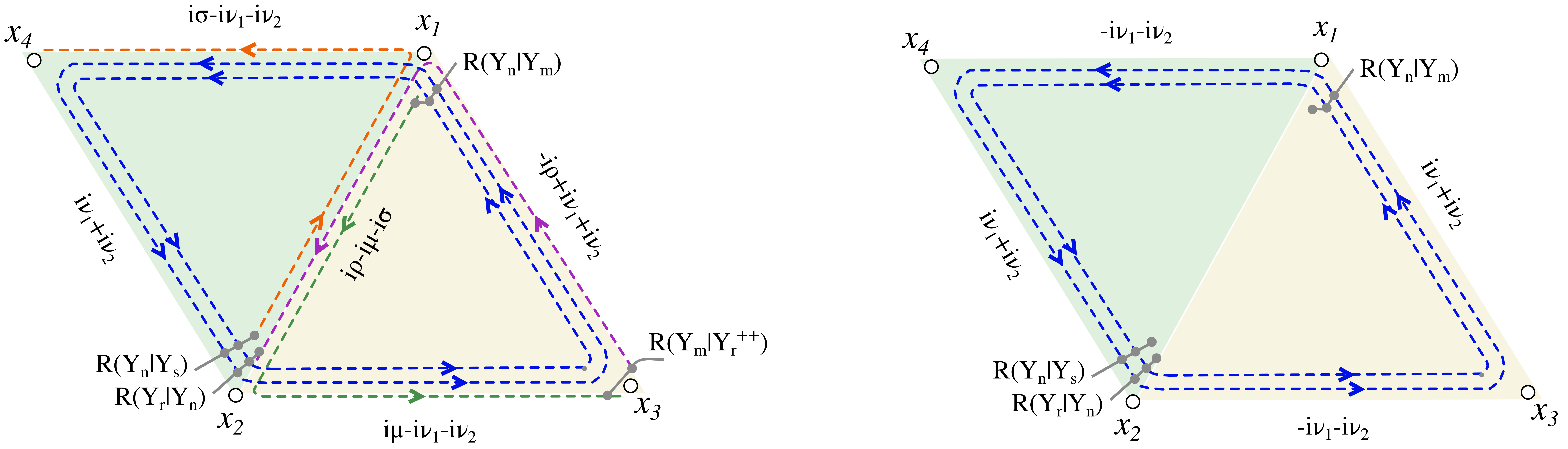
SOV REPRESENTATION OF DISK FISHNETS





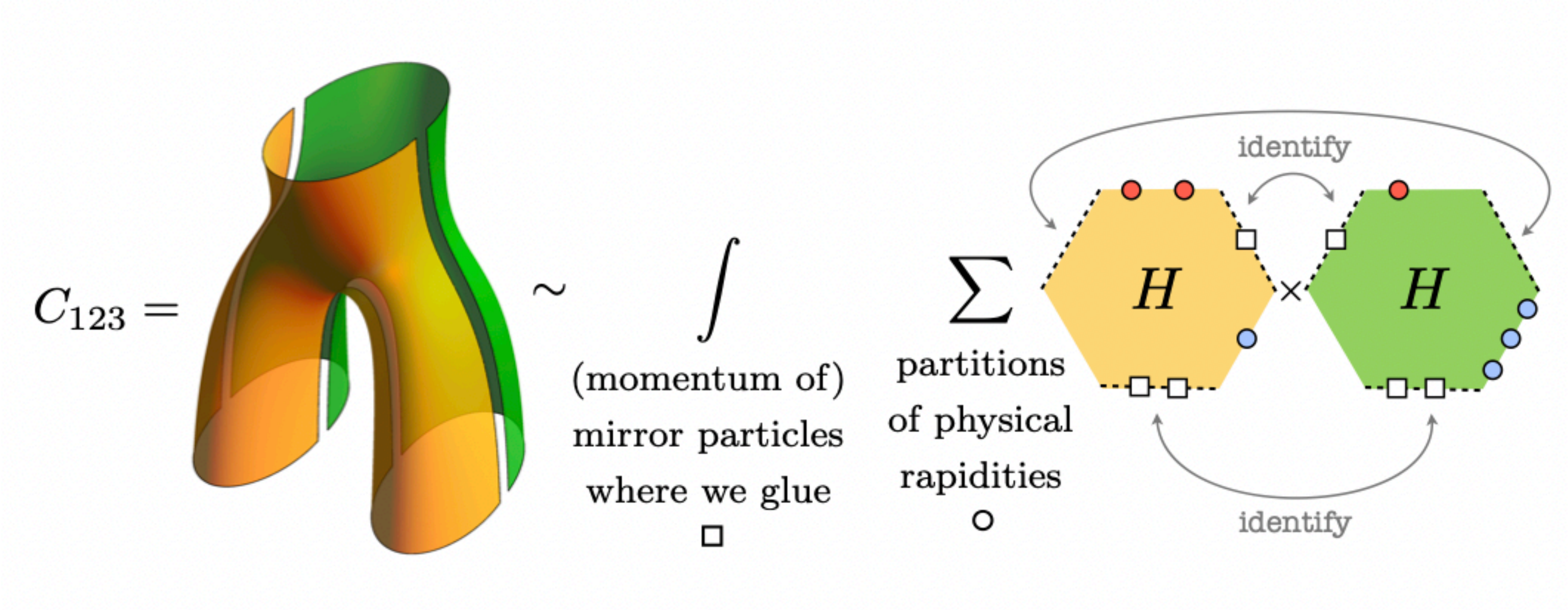
$$H(Y_n\,|\,Y_m)H(Y_n\,|\,Y_r)H(Y_s\,|\,Y_n)\times E(Y_{n_k})^L\times\frac{1}{H(Y_n\,|\,Y_n)}\times W(x_j,Y_{n_k})$$

SOV REPRESENTATION OF DISK FISHNETS

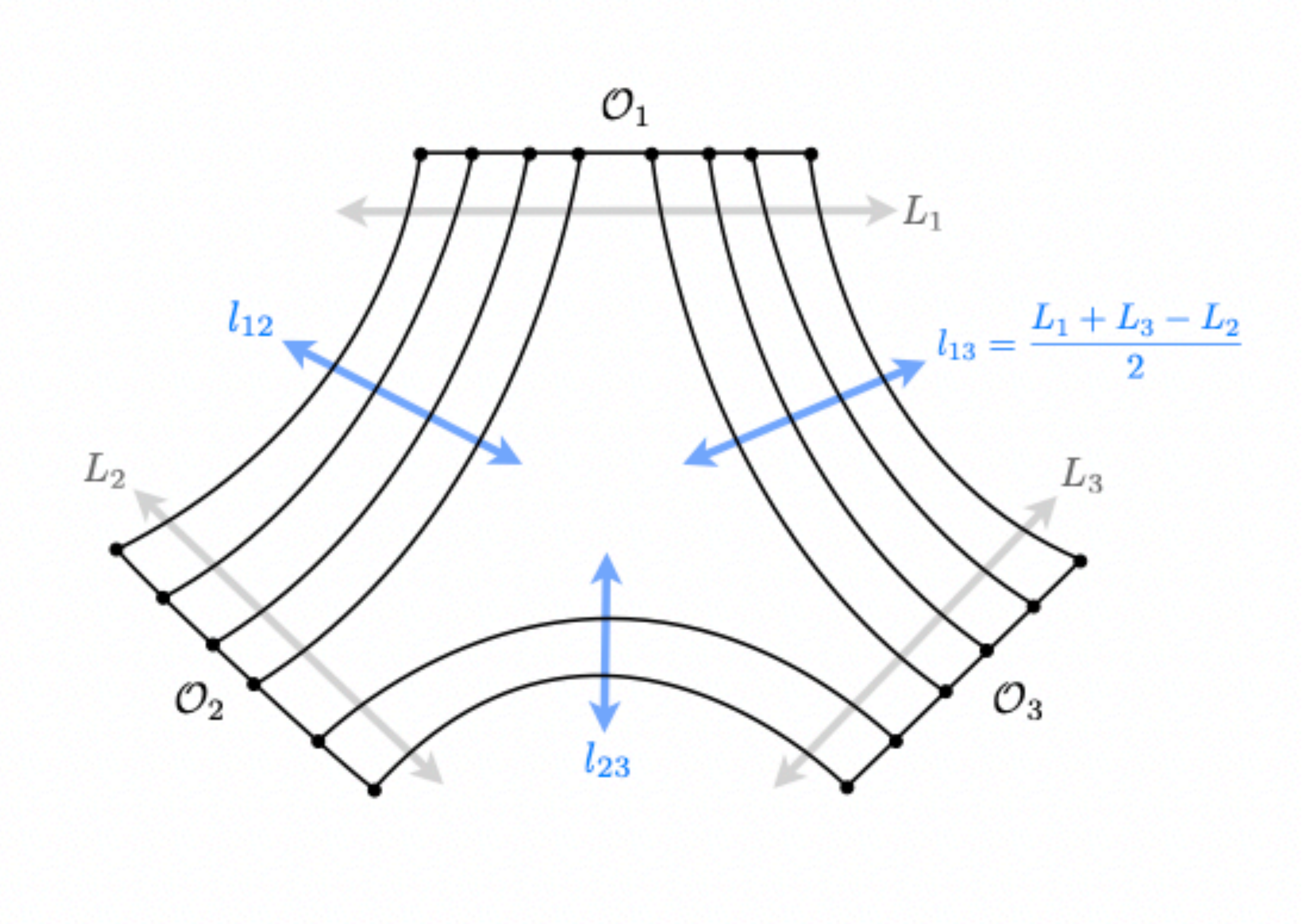


$$H(Y_n \mid Y_m) H(Y_n \mid Y_r) H(Y_s \mid Y_n) \times E(Y_{n_k})^L \times \frac{1}{H(Y_n \mid Y_n)} \times W(x_j, Y_{n_k})$$

FISHNET LIMIT OF N=4 SYM VS. FISHNET'S SPIN-CHAIN



[Basso, Sever, Vieira '15]

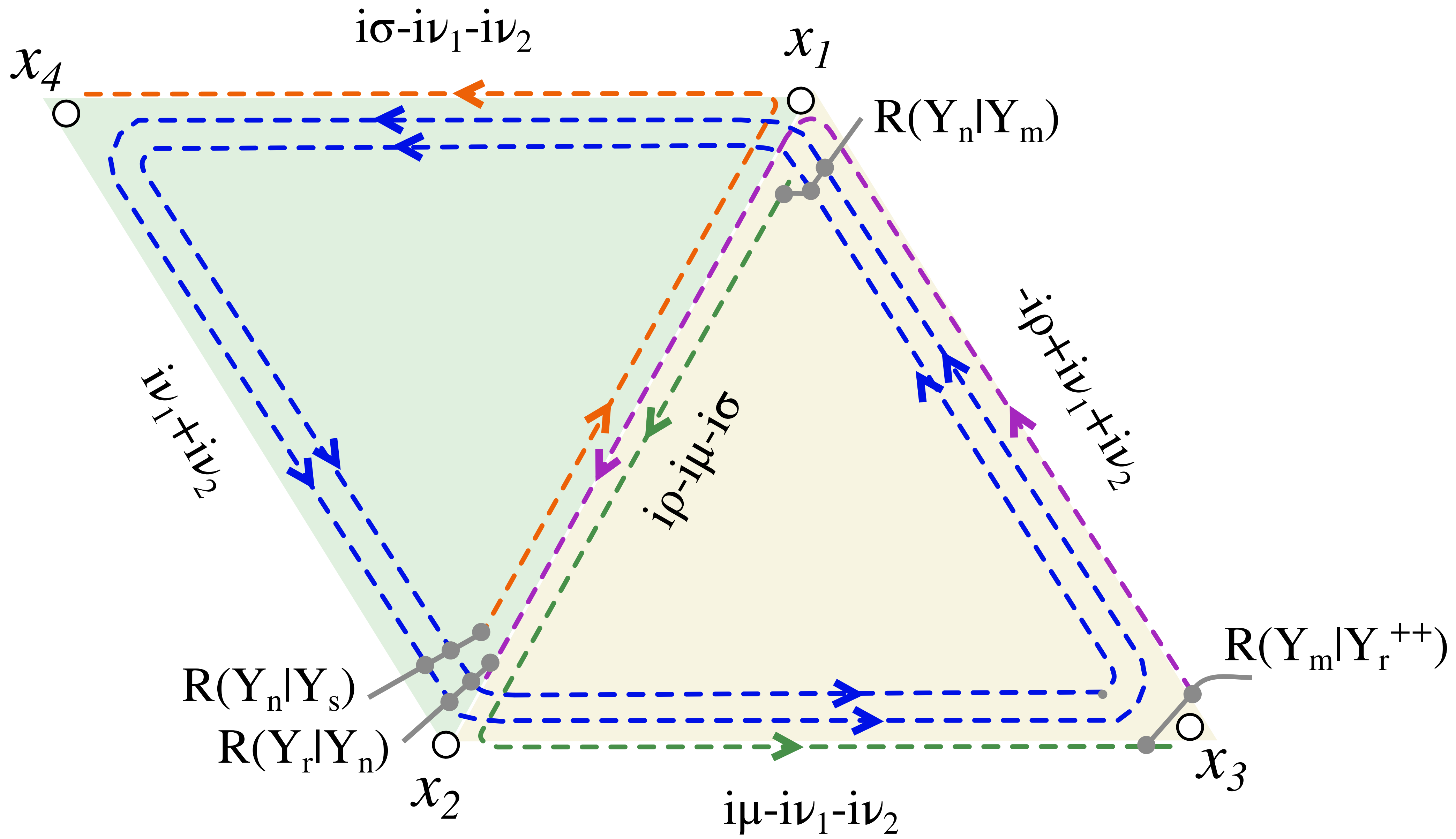


$$\mathcal{M}(\mathbf{u},\mathbf{v},\mathbf{w})|_{\text{amputated}} = \frac{c(\mathbf{v},\mathbf{w})}{c(\mathbf{u},\mathbf{w})} \times R(\mathbf{w}^{++},\mathbf{v})R(\mathbf{u},\mathbf{v})R(\mathbf{u},\mathbf{w}^{++})$$

$$H(\mathbf{u}\to\mathbf{v}|\mathbf{w}) = \frac{H_<(\mathbf{u},\mathbf{u})H_<(\mathbf{v},\mathbf{v})H_<(\mathbf{w},\mathbf{w})H(\mathbf{v},\mathbf{w})}{H(\mathbf{u},\mathbf{w})H(\mathbf{v},\mathbf{u})}$$

[Basso, Caetano, Fleury '17]

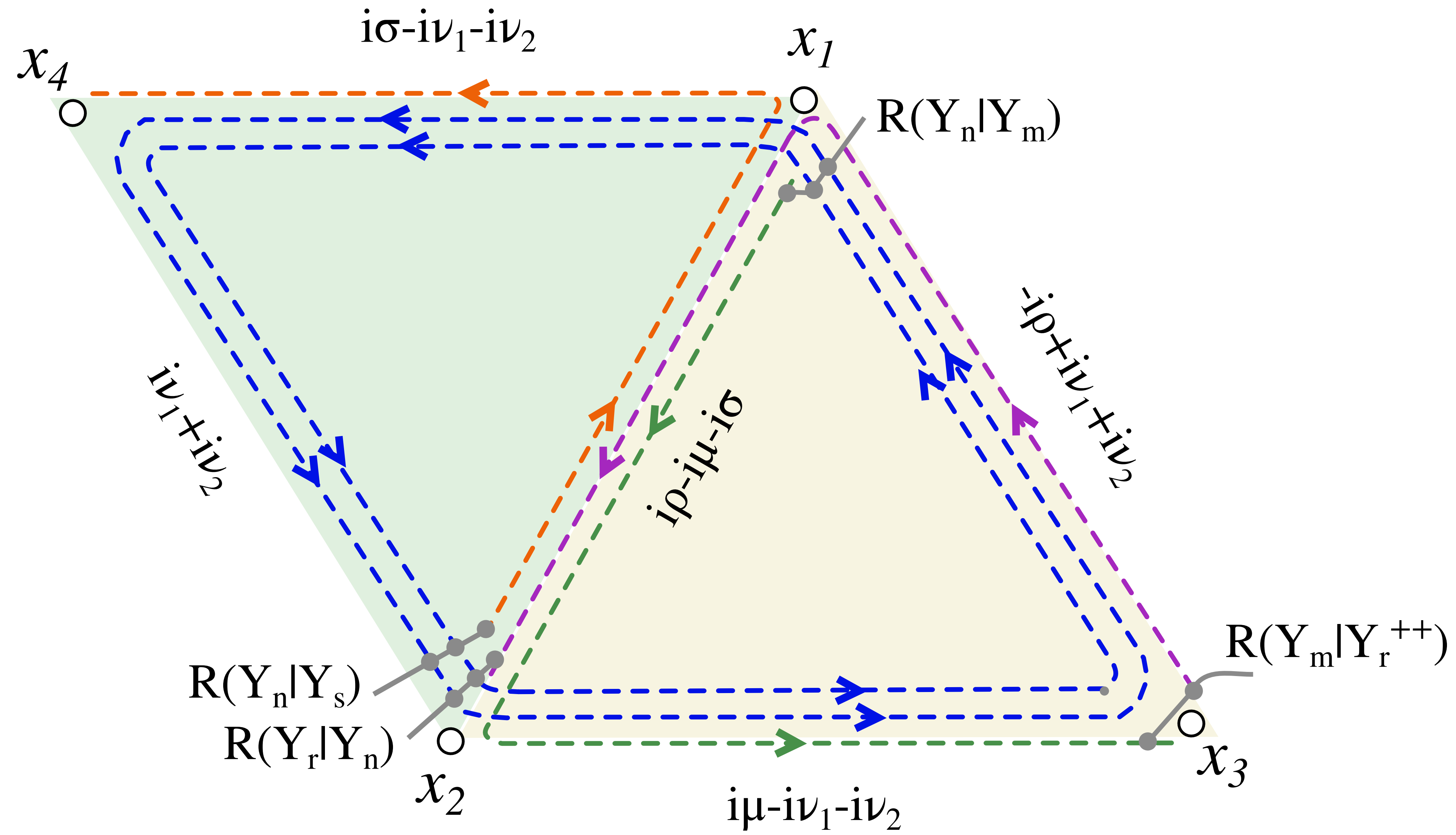
LIGHT-CONE LIMIT



$$\mathbf{R}^{bd}_{ac}(u) = \frac{1}{u+1}\left(u\,\delta^b_a\delta^d_c + \delta^d_a\delta^b_c\right) : \mathbb{C}^2 \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$\frac{[\bar{\mathbf{x}}_{12}]^{\mathbf{e}}_{\dot{\mathbf{b}}}\,\mathbf{R}^{\mathbf{bc}}_{\mathbf{ea}}(u)[\mathbf{x}_{12}]^{\dot{\mathbf{a}}}_{\mathbf{c}}}{(x_{12})^{2\beta}} = \frac{[\mathbf{x}_{12}]^{\dot{\mathbf{a}}}_{\mathbf{a}}\;[\overline{\mathbf{x}_{12}}]^{\mathbf{b}}_{\dot{\mathbf{b}}}}{(x_{12})^{2\beta}}$$

LIGHT-CONE LIMIT



- Simple representation for multi-loop Fishnet integrals in light-cone kinematics!
- Primer for type of functions describing multipoint Feynman integrals in 4d massless QFT.

$$\mathbf{R}^{bd}_{ac}(u) = \frac{1}{u+1}\left(u\,\delta^b_a\delta^d_c + \delta^d_a\delta^b_c\right) : \mathbb{C}^2\otimes\mathbb{C}^2 \longrightarrow \mathbb{C}^2\otimes\mathbb{C}^2$$

$$\frac{[\bar{\mathbf{x}}_{12}]^{\mathbf{e}}_{\dot{\mathbf{b}}}\,\mathbf{R}^{\mathbf{bc}}_{\mathbf{ea}}(u)[\mathbf{x}_{12}]^{\dot{\mathbf{a}}}_{\mathbf{c}}}{(x_{12})^{2\beta}} = \frac{[\mathbf{x}_{12}]^{\dot{\mathbf{a}}}_{\mathbf{a}}\,[\overline{\mathbf{x}_{12}}]^{\mathbf{b}}_{\dot{\mathbf{b}}}}{(x_{12})^{2\beta}}$$

Merci pour votre attention!