Topological gauging and non-relevant deformations of Quantum Field Theories

Based on work with S. Dubovsky and M. Porrati [2302.01654]

and work in progress with R. Borsato

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Les Diablerets

$$\mathscr{A} = \mathscr{A}_{\text{CFT}} + \mu \int d^D x\, \Phi_\Delta(x) + \sum_i^N \nu_i \int d^D x\, O_{d_i}(x) + \sum_i \alpha_i \int d^D x\, O_{\delta_i}(x)$$

RG fixed point

Relevant operator
$\Delta < D$

Marginal Operator
$d_i = D$

Irrelevant Operator
$\delta_i > D$

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(RG fixed manifold)

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(non-UV complete)

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Finite number

Infinite number

Wilson & Kogut, '74

$$\Sigma = \left\{ \mathscr{A}_\Lambda[\Phi] \,\middle|\, \mathscr{A}_\Lambda[\Phi] = \int_\Lambda d^D x\, \mathscr{L}[\Phi(x), \partial_\mu \Phi(x), \partial_\mu \partial_\nu \Phi(x), \ldots] \right\}$$

Space of "quasi-local" field theories

UV cutoff

Local density
(only depends on a space-time point x)

$\Downarrow$

non-locality range $< \epsilon = \Lambda^{-1}$

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Space of "quasi-local" field theories

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non-locality range $< \epsilon = \Lambda^{-1}$

On Σ, RG is a flow:

$$\frac{d}{d \log \epsilon} \mathscr{A}_\Lambda = B \left\{ \mathscr{A}_\Lambda \right\} \;, \quad B \left\{ \mathscr{A}_\Lambda \right\} \in T\Sigma \Big|_{\mathscr{A}_\Lambda}$$

$d \log \epsilon > 0 \;\Rightarrow$ IR, no pathology

$d \log \epsilon < 0 \;\Rightarrow$ UV, pathology!

Intrinsic UV scale Λ_*, $\mathscr{A}_\Lambda \notin \Sigma \,, \forall \Lambda > \Lambda_*$

$\Lambda_* = \infty \;\Rightarrow$ UV complete QFT


Σ
Here Be Dragons
Here Be Dragons
Here Be Dragons
CFT/Empty
?
CFT
O_{d_i}
CFT
Φ_Δ
O_{d_i}
$\tilde{\Phi}_\Delta$
$\Phi_\Delta + O_{d_i}$
O_{δ_i}
$T\overline{T}$
$T\overline{T}$
$T\overline{T}$
$-\log \Lambda_*$
Here Be Dragons

$\mathrm{T}\overline{\mathrm{T}}$ flow: $$\frac{\partial}{\partial\alpha}\mathscr{A}_\alpha[\Phi] = \int d^2x\, \underbrace{\varepsilon^{\mu\rho}\varepsilon^{\nu\sigma}T^{(\alpha)}_{\mu\nu}(x)T^{(\alpha)}_{\rho\sigma}(x)}$$

Smirnov, Zamolodchikov '16
Cavaglià, Negro, Szécsényi, Tateo '16

$$= \frac{1}{2}\det_{\mu\nu} T^{(\alpha)}_{\mu\nu} \propto T^{(\alpha)}\overline{T}^{(\alpha)} - \Theta^{(\alpha)\,2}$$

Well-defined through point-splitting

Important: $\partial^\mu T_{\mu\nu} = 0$

$T\overline{T}$ flow: $$\frac{\partial}{\partial\alpha}\mathscr{A}_\alpha[\Phi] = \int d^2x\, \underbrace{\varepsilon^{\mu\rho}\varepsilon^{\nu\sigma}T^{(\alpha)}_{\mu\nu}(x)T^{(\alpha)}_{\rho\sigma}(x)}$$

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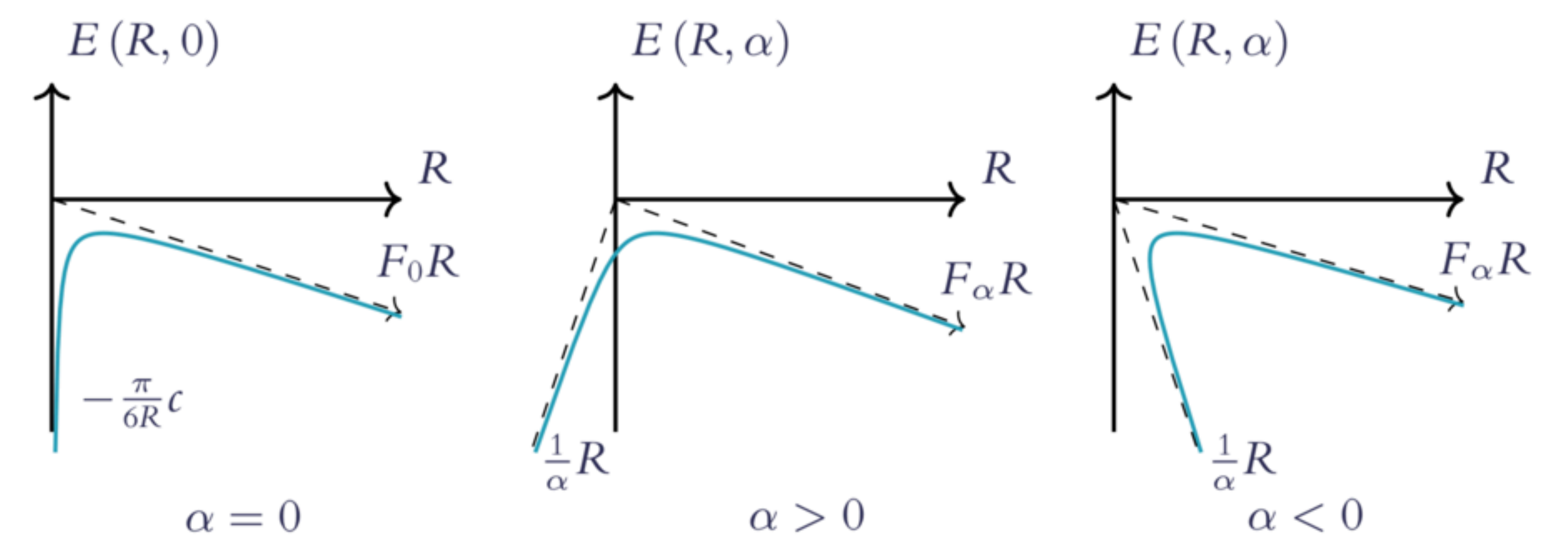
Important: $\partial^\mu T_{\mu\nu} = 0$

Irrelevant but controllable at all scales

Solvable, e.g. Burgers equation

$$\frac{\partial}{\partial\alpha}E_n(R,\alpha) + E_n(R,\alpha)\frac{\partial}{\partial R}E_n(R,\alpha) + \frac{1}{R}P_n(R)^2 = 0$$



Cavaglià, Negro, Szécsényi, Tateo '16

$\mathrm{T}\overline{\mathrm{T}}$ flow: $$\frac{\partial}{\partial\alpha}\mathscr{A}_\alpha[\Phi] = \int d^2x \; \underbrace{\varepsilon^{\mu\rho}\varepsilon^{\nu\sigma}T^{(\alpha)}_{\mu\nu}(x)T^{(\alpha)}_{\rho\sigma}(x)}$$

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Well-defined through point-splitting

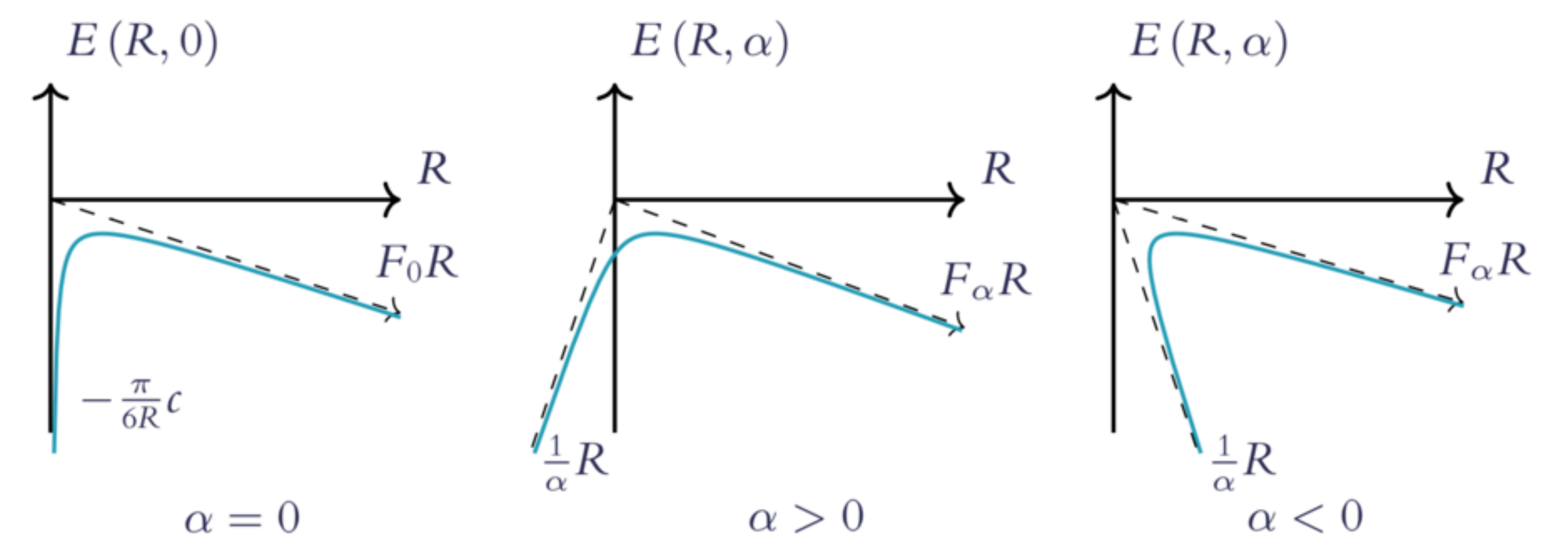
Important: $\partial^\mu T_{\mu\nu} = 0$

Irrelevant but controllable at all scales

Solvable, e.g. CDD deformation

$$S_\alpha(\theta) = e^{-i\alpha m^2 \sinh\theta} S_0(\theta)$$

Cavaglià, Negro, Szécsényi, Tateo '16
Dubovsky, Gorbenko, Mirbabayi '17



Universal in 2D

Connection to String Theory
(free boson $\xrightarrow[\mathrm{T}\overline{\mathrm{T}}]{}$ Nambu Goto)

Cavaglià, Negro, Szécsényi, Tateo '16

Connection to (Quantum) Gravity

Conti, Negro, Tateo '18

Dubovsky, Gorbenko, Mirbabayi '17 | Dubovsky, Gorbenko, Hernández-Chifflet '18

Generalisations →

CDD deformations $S = \Phi S_0\,, \qquad |\Phi|^2 = 1$

Al. Zamolodchikov '91
Mussardo, Simonetti '00
Dubovsky, Flauger, Gorbenko '12
Caselle, Fioravanti, Gliozzi, Tateo '13

|||

Double current deformations $\partial_\alpha \mathscr{A} = \int J \wedge J$

Conti, Negro, Tateo '19
Hernández-Chifflet, Negro, Sfondrini '20
Camilo, Fleury, Léncses, Negro, Zamolodchikov '21
Córdova, Negro, Schaposnik '21

This talk! Dubovsky, Negro, Porrati '23

Double-current flow: $\frac{\partial}{\partial\alpha}\mathscr{A}_\alpha[\Phi] = \int \underbrace{d^2x\ \varepsilon_{\mu\nu}\varepsilon^{ab} J_a^\mu(x\,|\,\alpha) J_b^\nu(x\,|\,\alpha)}$

Dubovsky, Negro, Porrati '22
(see also Cardy '19)

$= \varepsilon^{ab} J_a(\alpha) \wedge J_b(\alpha)$

$d^2x\,\partial_\mu J_a^\mu = d\star J_a = 0$

Well-defined through point-splitting

Double-current flow: $\frac{\partial}{\partial\alpha}\mathscr{A}_\alpha[\Phi] = \int \underbrace{d^2x\, \varepsilon_{\mu\nu}\varepsilon^{ab} J^\mu_a(x\,|\,\alpha) J^\nu_b(x\,|\,\alpha)}$

Dubovsky, Negro, Porrati '22
(see also Cardy '19)

$= \varepsilon^{ab}\, J_a(\alpha) \wedge J_b(\alpha)$

$d^2x\, \partial_\mu J^\mu_a = d \star J_a = 0$ Only true at $\alpha = 0$!

Well-defined through point-splitting

Double-current flow: $\frac{\partial}{\partial \alpha}\mathcal{A}_\alpha[\Phi] = \int \underbrace{d^2x\, \varepsilon_{\mu\nu} R^{ab} J_a^\mu(x\,|\,\alpha) J_b^\nu(x\,|\,\alpha)}$ $= R^{ab}\, J_a(\alpha) \wedge J_b(\alpha)$

Dubovsky, Negro, Porrati '22
(see also Cardy '19)

$d^2x\, \partial_\mu J_a^\mu = d \star J_a = 0$ True $\forall \alpha$ if $\omega = R^{-1}$ is a 2-cocycle

Well-defined through point-splitting

$\omega_{ab} = -\omega_{ba}\,,\qquad f_{ab}{}^{d}\omega_{cd} + f_{bc}{}^{d}\omega_{ad} + f_{ca}{}^{d}\omega_{bd} = 0$

$f_{ab}{}^{c}$: struct. const. of symmetry algebra

Double-current flow: $\frac{\partial}{\partial\alpha}\mathscr{A}_\alpha[\Phi] = \int \underbrace{d^2x\, \varepsilon_{\mu\nu} R^{ab} J_a^\mu(x|\alpha) J_b^\nu(x|\alpha)}$ $= R^{ab}\, J_a(\alpha) \wedge J_b(\alpha)$

Dubovsky, Negro, Porrati '22
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$\omega_{ab} = -\omega_{ba}$, $f_{ab}{}^d\omega_{cd} + f_{bc}{}^d\omega_{ad} + f_{ca}{}^d\omega_{bd} = 0$

$f_{ab}{}^c$: struct. const. of symmetry algebra

Define currents invariantly: $\mathscr{A}[\Phi] \longrightarrow \mathscr{A}[\Phi|A]$, $J_a^\mu(x) = -\frac{\delta}{\delta A_\mu^a(x)}\mathscr{A}[\Phi|A]\Big|_{A=0}$

Infinitesimally $\mathscr{A}_\alpha\left[\Phi\right] - \mathscr{A}_0\left[\Phi\right] = \alpha \int \varepsilon^{ab} J_a \wedge J_b + \mathcal{O}\left(\alpha^2\right)$

Hubbard-Stratonovich $e^{-\alpha \int \epsilon^{ab} J_a \wedge J_b} = \int [DA] e^{\frac{1}{4\alpha}\int \varepsilon_{ab} A^a \wedge A^b - \int J_a \wedge \star A^a}$

Current definition $\mathscr{A}_0\left[\Phi \,|\, \alpha A\right] = \mathscr{A}_0\left[\Phi\right] + \alpha \int J_a \wedge \star A^a + \mathcal{O}\left(\alpha^2\right)$

$\Longrightarrow \quad e^{-\mathscr{A}_\alpha[\Phi]} \sim \int [DA] e^{-\mathscr{A}_0[\Phi|A] + \frac{1}{4\alpha}\int \varepsilon_{ab} A^a \wedge A^b}$ Valid at first order in α

Quantum definition $\boxed{e^{-\mathscr{A}_\alpha[\Phi]} = \int [DA] e^{-\mathscr{A}_0[\Phi|A] + \frac{1}{4\alpha}\int \varepsilon_{ab} A^a \wedge A^b}}$ to all orders in α

Reproduces the flow equation classically

$$A^a = 2\alpha\varepsilon^{ab} \star J_b(\alpha) \, , \qquad J_a(\alpha) = -\frac{\delta}{\delta A^a}\mathscr{A}\left[\Phi | A\right]\Big|_{A^a = 2\alpha\varepsilon^{ab}\star J_b(\alpha)}$$

$$\mathscr{A}_\alpha\left[\Phi\right] \stackrel{?}{=} \mathscr{A}_0\left[\Phi | 2\alpha\varepsilon^{ab}\star J_b(\alpha)\right] - \alpha\varepsilon^{ab}\int J_a(\alpha)\wedge J_b(\alpha)$$

$$\frac{d}{d\alpha}\mathscr{A}_\alpha\left[\Phi\right] = 2\varepsilon^{ab}\int J_a(\alpha)\wedge\left(J_b(\alpha) + \alpha\frac{dJ_b(\alpha)}{\alpha}\right) - \varepsilon^{ab}\int J_a(\alpha)\wedge\left(J_b(\alpha) + 2\alpha\frac{dJ_b(\alpha)}{d\alpha}\right) = \varepsilon^{ab}\int J_a(\alpha)\wedge J_b(\alpha)$$

Quantum definition
$$e^{-\mathscr{A}_\alpha[\Phi]} = \int [DA\, DX] e^{-\mathscr{A}_0[\Phi|A] + \frac{1}{4\alpha}\int \varepsilon_{ab}(dX^a - A^a)\wedge(dX^b - A^b)}$$
to all orders in α

Stueckelberg fields

Gauge invariance

Reproduces the flow equation classically

$$A^a = 2\alpha\varepsilon^{ab} \star J_b(\alpha)\,, \qquad J_a(\alpha) = -\frac{\delta}{\delta A^a}\mathscr{A}\left[\Phi|A\right]\Big|_{A^a = 2\alpha\varepsilon^{ab}\star J_b(\alpha)}$$

$$\mathscr{A}_\alpha\left[\Phi\right] \stackrel{?}{=} \mathscr{A}_0\left[\Phi|2\alpha\varepsilon^{ab}\star J_b(\alpha)\right] - \alpha\varepsilon^{ab}\int J_a(\alpha)\wedge J_b(\alpha)$$

$$\frac{d}{d\alpha}\mathscr{A}_\alpha\left[\Phi\right] = 2\varepsilon^{ab}\int J_a(\alpha)\wedge\left(J_b(\alpha) + \alpha\frac{dJ_b(\alpha)}{\alpha}\right) - \varepsilon^{ab}\int J_a(\alpha)\wedge\left(J_b(\alpha) + 2\alpha\frac{dJ_b(\alpha)}{d\alpha}\right) = \varepsilon^{ab}\int J_a(\alpha)\wedge J_b(\alpha)$$

Quantum definition $\boxed{e^{-\mathscr{A}_\alpha[\Phi]} = \int [DA\, DX] e^{-\mathscr{A}_0[\Phi|A] + \frac{1}{4\alpha}\int \varepsilon_{ab}(dX^a - A^a)\wedge(dX^b - A^b)}}$ to all orders in α

In empty, featureless space-time

$$A^a = db^a\,, \qquad db^a = 2\alpha\varepsilon^{ab} \star J_b(\alpha)$$

Fix $X^a = 0$ (gauge choice)

$$\mathscr{A}_\alpha\left[\Phi\right] = \mathscr{A}_0\left[\Phi\,|\,db\right] - \frac{1}{4\alpha}\varepsilon_{ab}\int db^a \wedge db^b = \mathscr{A}_0\left[e^{-iq_a^{(\Phi)}\int db^a}\Phi\right]$$

$$\mathscr{Z}_\alpha = \int [D\Phi] e^{-\mathscr{A}_\alpha[\Phi]} = \int [D\Phi] e^{-\mathscr{A}_0\left[e^{-iq_a\int db^a}\Phi\right]} = \int [D\tilde{\Phi}] e^{-\mathscr{A}_0[\tilde{\Phi}]} = \mathscr{Z}_0$$

$$e^{-\mathscr{A}_\alpha[\Phi]} = \int [DA\,DX] e^{-\mathscr{A}_0[\Phi\,|\,A] + \frac{1}{4\alpha}\int \varepsilon_{ab}(dX^a - A^a)\wedge(dX^b - A^b)}$$

Suppose anomalies are present

$$[D\Phi] e^{-\mathscr{A}_0[\Phi|A]} \longrightarrow [D\Phi] e^{-\mathscr{A}_0[\Phi|A]}\, e^{C_{ab}\int \eta^a dA^b}$$

$$e^{-\mathscr{A}_\alpha[\Phi]} = \int [DA\, DX] e^{-\mathscr{A}_0[\Phi|A] + C_{ab}\int X^a dA^b + \frac{1}{4\alpha}\int \varepsilon_{ab}(dX^a - A^a)\wedge(dX^b - A^b)}$$

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$$e^{-\mathscr{A}_\alpha[\Phi]} = \int [DA\, DX] e^{-\mathscr{A}_0[\Phi\,|\,A] - C_{ab}\int X^a dA^b + \frac{1}{4\alpha}\int \varepsilon_{ab}(dX^a - A^a)\wedge(dX^b - A^b)}$$

Suppose anomalies are present

$$[D\Phi]e^{-\mathscr{A}_0[\Phi|A]} \longrightarrow [D\Phi]e^{-\mathscr{A}_0[\Phi|A]}\, e^{C_{ab}\int \eta^a dA^b}$$

Saddle point for Stueckelberg has singular points α_c

$$\left[C_{ab} - \frac{1}{2\alpha}\varepsilon_{ab}\right] dA^b = 0 \quad \Longrightarrow \quad 4\alpha_c^2 \mathrm{Det}(C) + 2\alpha_c \mathrm{Tr}(\varepsilon C) + 1 = 0$$

Example: compact boson $$\mathscr{A}_0[\varphi] = \frac{\beta^2}{2}\int d\varphi \wedge \star d\varphi \,, \quad \varphi \sim \varphi + 2\pi$$

Currents: shift and winding $$J_{\rm S} = \beta^2 d\varphi \,, \quad J_{\rm W} = \frac{1}{2\pi} \star d\varphi$$

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Minimal coupling $$\mathscr{A}_0[\varphi] = \int \left[\frac{\beta^2}{2}(d\varphi - B^{\mathrm{S}}) \wedge \star\, (d\varphi - B^{\mathrm{S}}) + \frac{1}{2\pi}(d\varphi - B^{\mathrm{S}}) \wedge B^{\mathrm{W}}\right]$$

Deformation easily found $$\mathscr{A}_\alpha[\varphi] = \frac{\beta(\alpha)^2}{2}\int d\varphi \wedge \star d\varphi\ , \quad \beta(\alpha) = \frac{\pi}{|\,\alpha - \pi\,|}\beta$$

Mixed anomaly! (shift B^{W})

$$C_{ab} = \begin{pmatrix} 0 & 0 \\ -\frac{1}{2\pi} & 0 \end{pmatrix}$$

Singular value

$$\mathrm{Det}\left[C - \frac{1}{2\alpha_c}\varepsilon\right] = 0 \implies \alpha_c = \pi$$

Minimal coupling

$$\mathscr{A}_0[\varphi] = \int \left[\frac{\beta^2}{2}(d\varphi - B^{\mathrm{S}}) \wedge \star\, (d\varphi - B^{\mathrm{S}}) + \frac{1}{2\pi}(d\varphi - B^{\mathrm{S}}) \wedge B^{\mathrm{W}}\right]$$

Deformation easily found

$$\mathscr{A}_\alpha[\varphi] = \frac{\beta(\alpha)^2}{2}\int d\varphi \wedge \star d\varphi\ , \quad \beta(\alpha) = \frac{\pi}{|\alpha - \pi|}\beta$$

Redefinition of fundamental fields $\Phi(x) \longrightarrow \tilde{\Phi}(\gamma_{x_0\to x}) = e^{-2\alpha i q_a \psi^a}\Phi(x)\,, \quad \psi^a = \varepsilon^{ab}\int_{\gamma_{x_0\to x}} \star J_b$

$\gamma_{x_0\to x}$ on constant-time slice and $x_0^1 = -\infty$

Fix constant s.t. $\psi^a(x) = \frac{1}{2}\varepsilon^a{}_b\left[Q^b_<(x) - Q^b_>(x)\right]$

- $Q^b_<(x)$: total b-charge to the left of x
- $Q^b_>(x)$: total b-charge to the right of x

} Operators!

Creation operators get dressed $a^\dagger_{\text{in}}(x_i) \longrightarrow e^{-i\alpha\varepsilon_{ab}q_i^a\left(Q^b_<(x_i) - Q^b_>(x_i)\right)}a^\dagger_{\text{in}}(x_i)$

In (and out) states get dressed $\left|\{x_i\}\right\rangle_{\text{in}} \longrightarrow e^{-i\alpha\varepsilon_{ab}\sum_{i<j}q_i^a q_j^b}\left|\{x_i\}\right\rangle_{\text{in}}$

$$\mathscr{A}_0\left[\phi\right] = \sum_{a=1}^{2}\int d^2x \left[\partial_\mu\phi_a^\dagger\partial^\mu\phi_a\right] = \int d^2x \left[\partial_\mu\rho_a\partial^\mu\rho_a + \rho_a^2\partial_\mu\theta_a\partial^\mu\theta_a\right] \,, \qquad \phi_a = \rho_a e^{i\theta_a}$$

Consider the phase rotation symmetries $J_\mu^a = i\left(\phi_a^\dagger\partial_\mu\phi_a - \phi_a\partial_\mu\phi_a^\dagger\right) = -2\rho_a^2\partial_\mu\theta^a$

Minimal coupling: $\mathscr{A}_0\left[\phi\,|\,A\right] = \sum_{a=1}^{2}\int d^2x \left[\left(\partial_\mu\phi_a + iA_\mu^a\phi_a\right)^\dagger\left(\partial_\mu\phi_a + iA_\mu^a\phi_a\right)\right]$

Solve $\frac{1}{2\alpha}\varepsilon_{ab}\varepsilon^{\mu\nu}A_\nu^b = -\frac{\delta}{\delta A_\mu^a}\mathscr{A}_0\left[\phi\,|\,A\right] = J_a^\mu - 2A_a^\mu\left|\phi_a\right|^2$

The result is a non-linear σ-model with B-field

$$\mathscr{A}_\alpha[\rho,\theta] = \int d^2x \left[\sum_{a=1}^{2} \left(\partial_\mu \rho_a \partial^\mu \rho_a + \frac{\rho_a^2 \partial_\mu \theta_a \partial^\mu \theta_a}{1 + 16\alpha^2 \rho_1^2 \rho_2^2} \right) - \frac{8\alpha \rho_1^2 \rho_2^2}{1 + 16\alpha^2 \rho_1^2 \rho_2^2} \varepsilon^{\mu\nu} \partial_\mu \theta_1 \partial_\nu \theta_2 \right]$$

It is a TST transformation of metric $ds^2 = d\rho_1^2 + d\rho_2^2 + \rho_1^2 d\theta_1^2 + \rho_2^2 d\theta_2^2$ Lunin, Maldacena '05

$(\theta_1, \theta_2) \xrightarrow{\text{T-duality}} (\tilde{\theta}_1, \theta_2)$

$(\theta_1, \theta_2) \xrightarrow{\text{TST}} (\theta_1, \theta_2 + \alpha\tilde{\theta}_1)$

$(\tilde{\theta}_1, \theta_2) \xrightarrow{\text{S-transform}} (\tilde{\theta}_1, \theta_2 + \alpha\tilde{\theta}_1)$

$(\tilde{\theta}_1, \theta_2 + \alpha\tilde{\theta}_1) \xrightarrow{\text{T-duality}} (\theta_1, \theta_2 + \alpha\tilde{\theta}_1)$

$\tilde{\theta}_1$: Obtained by dualising the radius of θ_1-circle

Buscher '88 | Rocek, Verlinde '92

the deformed system is a CFT.

$\Longrightarrow$ $\varepsilon^{ab} J_a \wedge J_b$ is a truly marginal operator, even though J_a are not properly CFT fields!

$$\frac{\partial}{\partial\alpha}\mathscr{A}_\alpha[\Phi] = \int d^2x\ \varepsilon_{\mu\nu}R^{ab}J_a^\mu(x\,|\,\alpha)J_b^\nu(x\,|\,\alpha)$$

Follow the same procedure to get

$$e^{-\mathscr{A}_\alpha[\Phi]} = \int [DA\,DX]e^{-\mathscr{A}_0[\Phi\,|\,A]+\frac{1}{2\alpha}\int \mathrm{Tr}\left[X\omega\left(\mathscr{D}A\right)\right]+\frac{1}{4\alpha}\int \mathrm{Tr}\left[A\wedge\omega(A)\right]}$$

These are Deformed T-Dual models   Borsato, Wulff '17

$\equiv$ to Yang-Baxter deformations of $\mathscr{A}_0[\Phi]$

generalise TST to non-abelian symmetry

$$\omega\left(\left[A,B\right]\right) = \left[\omega(A),B\right] + \left[A,\omega(B)\right]$$
$$\Downarrow\ R = \omega^{-1}$$
$$\left[R(A),R(B)\right] = R\left(\left[R(A),B\right] + \left[A,R(B)\right]\right)$$

Consider $\mathfrak{iso}(2)$: it allows a central extension $\left[P_a, P_b\right] = \omega_{ab} = \eta\varepsilon_{ab}\,,\qquad \left[K, P_a\right] = \varepsilon_a{}^b P_b$

Proceed as before in 1st order formalism $A_\mu = e^a_\mu P_a + \kappa_\mu K$

κ_μ → Spin connection

e^a_μ → Tetrad

The deforming term is JT gravity   Dubovsky, Gorbenko, Hernández-Chifflet '18

$$-\frac{\eta}{2\alpha}\int \varepsilon_{ab} dX^a \wedge e^b + \frac{\eta}{4\alpha}\int \varepsilon_{ab} e^a \wedge e^b = \frac{\eta}{4\alpha}\varepsilon_{ab}\int \left(dX^a - e^a\right) \wedge \left(dX^b - e^b\right)$$

$\mathrm{T\overline{T}}$ is ≡ to $\mathscr{A}_0$ on non-commutative Minkowski space-time

Same dressing of scattering matrix

Grosse, Lechner '07

Same correlation functions

Shyam, Yargic '22

To be done: fit the "higher $\mathrm{T}\overline{\mathrm{T}}$" deformations in this framework

Topological gauging of Virasoro subalgebras?
Higher spin gravity?

Extension to discrete and non-invertible symmetries $\Phi \to W\Phi$ → Wilson line

Extension to higher dimensions

Abelian seems possible: $\varepsilon_{a_1,\ldots,a_d} A^{a_1} \wedge \cdots \wedge A^{a_n}$
What about non-abelian?

Weak "conservation" breaking $d \star J = \varphi \,, \quad \left|\varphi\right| \ll 1$

Thank you

Higher-TTbar flow: $\frac{\partial}{\partial \alpha}\mathscr{A}_\alpha[\Phi] = \int \underbrace{d^2x\, \varepsilon^{\mu\rho}\varepsilon^{\nu\sigma}T^{(s)}_{\mu\nu}(x\,|\,\alpha)T^{(s)}_{\rho\sigma}(x\,|\,\alpha)}$ Hernandez-Chifflet, Negro, Sfondrini '19

Well-defined through point-splitting

$= \varepsilon^{\mu\nu} p^{(s)}_{1,\mu} p^{(s)}_{2,\nu}$

Factorized S-matrix known exactly: $S_\alpha(\theta) = e^{-i\alpha m^2/s\ \sinh s\theta} S_0(\theta)$ $\quad p^{(s)}_{j,\mu} = \left(\cosh s\theta_j\,,\ \sinh s\theta_j\right)$

Higher-TTbar flow: $\frac{\partial}{\partial\alpha}\mathscr{A}_\alpha[\Phi] = \int \underbrace{d^2x\, \varepsilon^{\mu\rho}\varepsilon^{\nu\sigma}T^{(s)}_{\mu\nu}(x\,|\,\alpha)T^{(s)}_{\rho\sigma}(x\,|\,\alpha)}_{\text{Well-defined through point-splitting}}$ Hernandez-Chifflet, Negro, Sfondrini '19

Factorized S-matrix known exactly: $S_\alpha(\theta) = e^{-i\alpha m^2/s\ \sinh s\theta}S_0(\theta)$ (sinh $s\theta$ → Higher-spin charge)

$\Downarrow$

Access to TBA equations. Kernel: $\varphi_\alpha(\theta) = \frac{1}{2\pi i}\frac{d}{d\theta}\log S_\alpha(\theta) = \varphi_0(\theta) - \frac{\alpha m^2}{2\pi}\cosh s\theta$

$$\varepsilon(\theta) = \underbrace{r\cosh\theta}_{\text{Driving term}} - \int d\theta'\,\varphi_\alpha(\theta-\theta')\log[1+e^{-\varepsilon(\theta')}]$$

$$\mathscr{I}^\pm_s = \int d\theta\, e^{\pm s\theta}\log[1+e^{-\varepsilon(\theta)}]$$

$$\varepsilon(\theta) = \underbrace{r\cosh\theta + \frac{\alpha m^2}{\pi}\left(\mathcal{J}_s^+ + \mathcal{J}_s^-\right)\cosh s\theta}_{} - \int d\theta'\,\varphi_0(\theta-\theta')\log[1+e^{-\varepsilon(\theta')}]$$

Modified driving term ⟶ Different thermal bath ≡ Twisted boundary conditions

$$\varepsilon(\theta) = \underbrace{r\cosh\theta}_{} - \int d\theta'\,\varphi_\alpha(\theta-\theta')\log[1+e^{-\varepsilon(\theta')}]$$

Driving term

$$\mathcal{J}_s^\pm = \int d\theta\, e^{\pm s\theta}\log[1+e^{-\varepsilon(\theta)}]$$

$$\varepsilon(\theta) = \underbrace{r\cosh\theta + \frac{\alpha m^2}{\pi}\left(\mathcal{J}_s^+ + \mathcal{J}_s^-\right)\cosh s\theta}_{} - \int d\theta'\,\varphi_0(\theta-\theta')\log[1+e^{-\varepsilon(\theta')}]$$

Modified driving term ⟶ Different thermal bath ≡ Twisted boundary conditions

Generalize: Consider an arbitrary superposition of deformations

$$\frac{\partial}{\partial\alpha_s}\mathscr{A}_{\boldsymbol{\alpha}}[\Phi] = \int d^2x\,\varepsilon^{\mu\rho}\varepsilon^{\nu\sigma}T^{(s)}_{\mu\nu}(x\,|\,\boldsymbol{\alpha})T^{(s)}_{\rho\sigma}(x\,|\,\boldsymbol{\alpha})$$

$$\varepsilon(\theta) = \underbrace{\sum_s\left[\gamma_s(\mathcal{J})\,e^{s\theta} + \gamma_{-s}(\mathcal{J})\,e^{-s\theta}\right]}_{\text{GGE}} - \int d\theta'\,\varphi_0(\theta-\theta')\log[1+e^{-\varepsilon(\theta')}]$$

Knowledge of the undeformed GGE TBA yields results for the deformed theory

$$\left[\frac{\partial}{\partial \alpha_s} + \mathcal{I}_{-s}(\boldsymbol{\gamma},\boldsymbol{\alpha})\frac{\partial}{\partial \gamma_s} + \mathcal{I}_s(\boldsymbol{\gamma},\boldsymbol{\alpha})\frac{\partial}{\partial \gamma_{-s}}\right]\mathcal{I}_t(\boldsymbol{\gamma},\boldsymbol{\alpha}) = 0 \quad \Longleftrightarrow \quad \mathcal{I}_t(\boldsymbol{\gamma},\boldsymbol{\alpha}) = \mathcal{I}_t(\boldsymbol{\gamma} + \boldsymbol{\alpha}\boldsymbol{\mathcal{I}}(\boldsymbol{\gamma},\boldsymbol{\alpha}),\mathbf{0})$$

Hernández-Chifflet, Negro, Sfondrini '20

Generalize: Consider an arbitrary superposition of deformations

$$\frac{\partial}{\partial \alpha_s}\mathcal{A}_{\boldsymbol{\alpha}}\left[\Phi\right] = \int d^2x\ \varepsilon^{\mu\rho}\varepsilon^{\nu\sigma}T^{(s)}_{\mu\nu}(x\,|\,\boldsymbol{\alpha})T^{(s)}_{\rho\sigma}(x\,|\,\boldsymbol{\alpha})$$

$$\varepsilon(\theta) = \underbrace{\sum_s \left[\gamma_s\left(\mathcal{I}\right)e^{s\theta} + \gamma_{-s}\left(\mathcal{I}\right)e^{-s\theta}\right]}_{\text{GGE}} - \int d\theta'\,\varphi_0(\theta-\theta')\log[1+e^{-\varepsilon(\theta')}]$$